Determinants of Operating Expenses in Massachusetts Affordable Multifamily Rental Housing

Prepared for Massachusetts Housing Partnership

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Executive Summary

Operating expense levels vary greatly among affordable multifamily properties, and the ability to predict a project's operating expenses is critical to lenders and owners alike in order to establish a sustainable financing structure. What factors are responsible for the variation in operating expenses across properties? This study employs regression analysis to quantify the impacts of various project characteristics on operating expenses. The objectives of this analysis are to:

1) Inform operating expense forecasting in the underwriting process;
2) Help establish a benchmarking tool for operating properties; and
3) Identify areas for further study in order to guide future data collection and research.

Using data for 625 Massachusetts affordable rental housing properties supplied by Massachusetts Housing Partnership (MHP) and Massachusetts Housing Finance Agency (MassHousing), this analysis confirms that a number of project characteristic variables have a relationship with operating expense levels. Predicting variation in operating expenses and precisely quantifying the relationship between project characteristics and operating expense was, however, more challenging than initially anticipated. While the results do not deliver the precision required to add immediate value to underwriting or benchmarking processes, this analysis provides a strong foundation for future study by identifying project characteristics worthy of further examination and revealing patterns in the data that can direct subsequent research design.

Variables identified as being associated with an increased level of operating expenses per unit include:

- *Average Bedrooms Per Unit*: As average bedrooms per unit increase, costs across several expense categories, including administrative and management fees, maintenance, water, and insurance, increase on a per unit basis.
- *LIHTC Status:* Projects financed with Low Income Housing Tax Credits (LIHTC) appears to be associated with increased operating expense levels, though there is a lack of consistency across portfolio data sets. Administrative and management fees and utilities are higher for LIHTC projects.
- *Percent Project-Based Section 8:* Administrative and management fees, maintenance, and services all increase as the proportion of Project-Based Section 8 units increases.
- *Boston Market Location:* Projects located in the City of Boston appear to be more expensive to operate due to higher utility and security costs per unit. Projects located in the Boston Market of Brookline, Cambridge, and Somerville are slightly less costly than projects in Boston, but also show elevated levels of administrative expense and management fees, maintenance, utilities, water, services, security, and real estate taxes as compared to projects in other jurisdictions.
- *Presence of Services and/or Security:* As would be expected, presence of services and security respectively are both associated with increased levels of operating expenses per unit.

Variables identified as being associated with a decreased level of operating expenses per unit include:

- *Number of Buildings:* Each additional building is associated with a decrease in per unit operating expenses, though no specific category within operating expenses could be traced as the source of this relationship.
- *SRO Status:* Several operating expense categories tend to be lower for SRO projects: management fees, maintenance costs, services, and insurance.
- *Gateway City Location:* Projects located in Gateway Cities appears to be have lower management fees, maintenance, real estate and insurance expense per unit.

The relationship with the following independent variables and operating expenses per unit was inconclusive:

- *Number of Units:* Analysis of individual operating expenses indicates that some expenses may decrease on a per unit basis as number of units increases, but others appear to increase.
- *Percent Affordable:* Findings are inconclusive as to the impact of the proportion of affordable units on operating expense levels, as the MHP and MassHousing data sets show inverse relationships with operating expenses per unit for this variable.
- *Primary Program Type (Interest Subsidy, Project-Based Section 8, SHARP/RDAL, and Other):* No consistent, statistically significant relationship between overall expenses per unit and primary program type is seen in the regression models. Limited associations were, however, identified related to individual expense categories. SHARP/RDAL projects show lower administrative and management expenses and lower service costs but higher maintenance expense per unit than other projects. Both utilities and water per unit costs are higher for Interest Subsidy, Section 8 and SHARP/RDAL projects as compared to other projects.

Differences in the composition of projects in the two agency's portfolios create significant variation in findings across the data sets. There are two possible reasons for this variation:

1) The effect of some variables on operating expense levels may vary depending on other project characteristics.
2) There may be additional key variables that have bearing on operating expenses that are not included in the analysis, the omission of which distorts findings.

The discovery of inconsistencies in regression results across data sets, which was made possible by the availability of data from two different agencies, is valuable for guiding future research and data collection efforts.

Introduction[1]

Controlled and predictable operating expenses are one of the most important factors in the long-term financial health of a multifamily rental project. Maintaining a supportable level of expenses ensures the ability to meet debt service obligations, retain a healthy cash flow, and accurately budget for the long-term needs of a property. For these reasons, operating expenses are key inputs in the initial structuring of a project's financing. As a component of underwriting, expense predictions help to determine the maximum amount of debt the property can carry, and therefore the amount of subsidy that is needed to make the deal viable. The ability to predict operating costs as accurately as possible is mutually beneficial to borrowers and lenders, as increased certainty translates to a decrease in risk.

Among other factors, it is argued that housing quality, property size, building systems, geography/location, management practices, and the involvement of various affordable housing programs impact a project's operating expenses. This study uses regression analysis to quantify the impacts of these project characteristics on operating expenses. This analysis was commissioned with a threefold purpose. First, the work was hoped to provide a reference when forecasting operating expenses in the underwriting process. Next, results were expected to help establish benchmarking tools for properties already in operation. Last, this analysis was intended to identify areas for future study to guide data collection efforts and research regarding the determinants operating expenses.

The results of this study illuminate the complexity of the task of predicting of operating expenses. Though the findings do not provide the predictive precision needed to benefit underwriting or benchmarking processes without additional analysis, this work contributes strong groundwork from which future research can build.

This work builds on initial analysis undertaken by MHP of its portfolio that preliminarily identified a small group of project characteristics that predict operating expenses. An expanded group of likely determinants of operating expenses was identified to guide the design of this project by the Real Estate Finance Working Group, a group of affordable housing professionals chaired by MHP and Massachusetts Association of Community Development Corporations staff. MHP partnered with MassHousing to collect data for analysis. The provision of data from both lenders was intended to maximize predictive precision by increasing the sample size, as well as to maximize the external validity of results.

Overview of Project Design and Methodology

The primary data set used for this analysis included portfolio data from both MHP and MassHousing, totaling 625 properties. The data included operating expense information by category for fiscal years 2008 and 2009, as well as additional project data. A detailed list and definitions of the variables included in the data is included in Appendix A. The following independent variables were included in the combined data set:

[1] This section was partially authored by Massachusetts Housing Partnership.

- Number of Units
- Average Bedrooms Per Unit
- Number of Buildings
- LIHTC Status
- Percent Project-Based Section 8
- Percent Affordable
- Primary Program Type (Interest Subsidy Program, Section 8 Program, SHARP/RDAL, Other)
- SRO Status
- Boston Market Location (Boston, Brookline, Cambridge, and Somerville)
- City of Boston Location
- Gateway City Location
- Presence of Services
- Presence of Security

The dependent variables examined included:

- Total Operating Expenses Per Unit (net of replacement reserve contributions)
- Administrative Expense and Management Fee Per Unit
- Maintenance Expense Per Unit
- Utilities Per Unit (net of water expense)
- Water Per Unit
- Services Per Unit
- Security Per Unit
- Real Estate Taxes Per Unit
- Insurance Per Unit

Multiple regressions were run to test whether the independent variables have bearing in determining predicted operating expenses levels and to identify the magnitude of this relationship. Regressions completed for the combined data set were also completed for the MHP and MassHousing data sets individually. This step proved valuable for identification of areas where results may be distorted due to the omission of other determinants of operating expenses not included in the analysis.[2]

In addition, individual regressions for each the MHP and MassHousing data sets containing additional available variables were undertaken. This analysis is intended to identify possible relationships with additional independent variables to guide future data collection and research.

Additional variables examined for the MHP portfolio included: [3]

- Age of Property
- Rehab/New Construction

[2] This topic is discussed in detail on p. 8.
[3] A list of the data supplied by each agency and variable definitions can be found in Appendix A.

- Years since Rehab
- Property Condition
- Average Unit Area
- Management Quality
- Self-Managed/Third-Party Managed
- Non-Profit Developer/For-Profit Developer
- Percent Single-Room Occupancy
- Vacancy Rate

Additional variables examined for the MassHousing portfolio included:

- Physical/Management PMR
- Number of Elevators
- Scattered Site
- Heat Individually/Master-Metered
- Electricity Individually/Master-Metered
- Heat Type
- 4% LIHTC/9% LIHTC
- Percent Elderly

Findings

The regressions performed confirm that a number of project characteristic variables have a relationship with operating expense levels. Further, study of individual operating expense categories provides insight as how and why given project characteristics are associated with a higher or lower level of operating expenses.

Predicting variation in operating expenses and quantifying the relationship between project characteristics and operating expense levels was, however, more challenging than initially anticipated. The best regression models employed for the combined data set predict about half of the variation in operating expenses.[4] Differences in the composition of projects in the two agency's portfolios create significant variation in magnitude of the associations across the data sets. This indicates that there are additional key variables that have bearing on operating expenses but are not included in the analysis. A summary of the findings related to each independent variable follows. Next, likely reasons for the differences in findings across agency portfolio data sets are outlined, and the implications of these differences are discussed.

Summary of Independent Variable Findings[5]

The following summary draws on the best predictive fit regression model for the combined data set (included in Appendix C) to highlight associations identified between

[4] Appendix D includes histograms which provide a visual representation of 1) the distribution of operating expenses per unit across the projects in the data set, and 2) accuracy of the best predictive regression model at predicting individual project operating expenses.

[5] A more detailed version of this summary can be found in Appendix G.

the various project characteristics and overall operating expenses per unit. It also utilizes regressions performed with operating expense categories as the dependent variable (included in Appendix F) to illuminate more specifically what expense items are impacted by a given project characteristic.

Number of Units: Isolating a relationship between number of units and operating expenses proved surprisingly difficult. Several forms of independent variables representing number of units (Number of Units as a continuous variable, Log Number of Units, and other dummy variables for unit range categories) were tested, and the categories 0-20 units, 21-60 units, and 60+ units provided the strongest predictive power but were not jointly statistically significant. Analysis of individual operating expenses indicates that some expenses may decrease on a per unit basis as number of units increases, but others appear to increase. Insurance expense per unit, for instance, appears to be most costly for smaller properties, whereas services and security expense per unit escalate as property size increases.

Average Bedrooms Per Unit: The greater the number of average bedrooms per unit, the higher operating expenses per unit. The best way to model this variable utilizes a quadratic functional form. This form is employed because the data indicates that an increase from zero to one average bedrooms per unit relates to a smaller operating expense per unit delta as compared to an increase from one to two average bedrooms per unit. This pattern holds true as average bedrooms increase: the jump from two to three average bedrooms and three to four average bedrooms are larger than the increase from one to two average bedrooms or two to three average bedrooms respectively.[6] As average bedrooms per unit increase, costs across several operating expense categories increase on a per unit basis. These include administrative and management fees, maintenance, water, and insurance.

Number of Buildings: The data indicates that each additional building is associated with a lower level of per unit operating expenses. A quadratic functional form is used because the magnitude of the change in operating expenses associated with one additional building decreases slightly the higher the number of buildings. This finding is similarly present in the MassHousing data; the MHP data, however, shows a much smaller magnitude for the relationship between number of buildings and operating expenses that is not statistically significant. There are no categories within operating expenses that showed an association to number of buildings of a material magnitude.

LIHTC Status: Projects financed with LIHTC may be associated with increased operating expense levels, though there is a lack of consistency across the portfolio data sets. A larger and statistically significant elevation in expenses associated with LIHTC status is seen in the MHP data while the MassHousing data shows a small, non-statistically significant correlation. These disparate results indicate that distinct characteristics common to each portfolio that are not included as control variables create

[6] Additional discussion of quadratic functional form is provided in Appendix B. Appendix C provides the precise associated increase in expenses per unit that is indicated by the data for each average bedroom per unit size and for other variables for which quadratic functional form is used.

biases. While the magnitude of the increase associated with LIHTC status is difficult to quantify from available data, analysis of categories reveals some insight as to why LIHTC status appears correlated with elevated per unit operating expense levels. Administrative and management fees and utilities are higher for LIHTC projects than non-LIHTC projects.

Percent Project-Based Section 8: The greater the proportions of Project-Based Section 8 units, the higher the expected operating expense per unit. A quadratic model provides the best fit to describe this relationship. As the proportion of Project-Based Section 8 units increases, the impact of an incremental increase in Section 8 units becomes larger in magnitude. The relationship of Project-Based Section 8 units and operating expenses is seen across several operating expense categories. Administrative and management fees, maintenance, and services all increase as the proportion of Project-Based Section 8 units increases.

Percent Affordable: There is no statistically significant relationship between the proportion of affordable units and operating expenses apparent in the combined data or the MHP data. It is possible, however, that findings are distorted due to distinctions in characteristics between the two agency portfolios that are not included as control variables. The MassHousing data does show a statistically significant decrease in per unit expense levels for each percentage point increase in affordable units. There is only one operating expense category where a statistically significant relationship to percent affordable was found: real estate taxes appear to increase modestly as the proportion of affordable units increases.

Primary Program Type: No consistent, statistically significant relationship between overall expenses per unit and primary program type is seen in the regression models. The inclusion of program type does, however, notably alter the magnitude of other independent variable coefficients, making the MHP and MassHousing data set coefficients more similar, which indicates that controlling for program type is useful for accurately isolating the association between various independent variables and expenses. A few expense categories show a statistically significant relationship with one or more program types. SHARP/RDAL projects show lower administrative costs and management fees and lower service costs but higher maintenance expense per unit than other projects. Both utilities and water per unit costs are higher for Interest Subsidy, Section 8 and SHARP/RDAL projects as compared to other projects.

SRO Status: SROs are considerably cheaper to operate than other properties on a per unit basis. On top of the finding previously discussed that per unit operating expenses increase as average bedroom per unit increases, SRO projects are associated with a much lower level of operating expenses per unit, which is seen consistently in the MHP and combined data sets.[7] It should be noted that there are no SROs in the MassHousing portfolio so all SROs in the combined data set are MHP projects. Several operating

[7] The SRO coefficient indicates the predicted difference in operating expenses per unit for an SRO project as compared to a hypothetical baseline project with zero average bedrooms per unit.

expense categories tend to be lower for SRO projects: management fees, maintenance costs, services, and insurance.

Location: Projects located in the Boston Market of Boston, Brookline, Cambridge, and Somerville are associated with an elevated level of operating expenses per unit. These projects have higher per unit costs in administrative expense and management fees, maintenance, utilities, water, services, security, and real estate taxes. The data indicates that location in the City of Boston itself may be associated with additional elevated costs above the Boston Market levels. However, the data sets do not show a consistent or statistically significant distinction. Utility costs and security cost appear higher for the City of Boston proper as compared to Brookline, Cambridge, and Somerville, while service costs appear lower in the City of Boston as compared to these neighboring jurisdictions.

Location in a Gateway City appears to be associated with a decreased level of operating expenses per unit, though the relationship is less definitive than that of the Boston Market variable. The expense categories where Gateway City Location appear statistically significantly lower include management fees, maintenance, real estate and insurance. Security cost, however, appears to be higher for properties in Gateway Cities.

Presence of Services: As would be expected, presence of services at a property is associated with an increase in operating expenses and inclusion of this variable improves the predictive power of the regressions. High variation in cost, however, makes the magnitude of this increase difficult to predict.

Presence of Security: Again, inclusion of the Security variable improves the predictive power of the regression, but magnitude is unclear and coefficients are not statistically significant for the combined data or the MHP data.

Discrepancy of Findings Across Data Sets: Implications for Future Research

Though it may be initially puzzling to see variation in regression coefficients for the same independent variables across the MHP and MassHousing data sets respectively, this finding is very useful for guiding future research. There are two potential reasons for the discrepancies in the regression models:

1) The effect of some variables on operating expense levels may vary depending on other project characteristics.
2) There may be additional key variables that have bearing on operating expenses that are not included in the analysis, the omission of which distorts findings.

First, the effect of some variables on operating expense levels may vary depending upon other project characteristics. In other words, subpopulations of projects that share a single characteristic or combination of characteristics may experience disparate effects on operating expenses associated with other variables. As shown in Appendix G, MHP and MassHousing's portfolios have different compositions in terms of size, affordability proportion, project financing, and other characteristics. If, for example, operating

expenses per unit for projects over 60 units were affected differently than projects under 60 units by the number of buildings at a property, then the MassHousing regression, which has a greater proportion of projects over 60 units, would show a different coefficient on the variable Number of Buildings than the MHP regression.[8] It may be that the regression coefficients vary between the two data sets because the best predictive regression models are different for each portfolio due to their distinctive compositions.

Regression models can allow the amount of change in the dependent variable (in this case, Operating Expenses Per Unit) that is associated with an incremental unit of a given independent variable to vary for different subpopulations of projects. This requires the use of interaction variables, which are independent variables that represent the product of two or more independent variables. Future research that examines whether the interaction of pairs or groups of independent variables are statistically significant would be beneficial in order to confirm whether there are different effects of independent variables on different subpopulations of projects. Regressions that include statistically significant interactions have the potential to be much stronger at predicting variation in operating expenses across projects.

Second, it is possible that the existence of additional key project characteristics that are not included as independent variables may bias results. This issue is referred to as omitted variable bias. Defined in econometric language, omitted variable bias is a situation in which an independent variable that is 1) a determinant of the dependent variable, and 2) correlated with a second independent variable, is excluded from a regression, resulting in distortion of the coefficient on the second independent variable. Another way to think of this issue is that regressions must control for any independent variables that have an association with other independent variables in order to measure the true relationship of the independent variables to the dependent variable. If such control variables are not included, the regression coefficients on the independent variables will include a portion of the relationship between the missing variables and the dependent variable.

The concept of omitted variable bias may best be explained through a practical example, as follows. The Real Estate Working Group hypothesized that the proportions of elderly units could have an impact on operating expenses per unit. Elderly units tend to be smaller than family units, and unit size is another independent variable that was hypothesized to potentially impact per unit operating expense levels. To determine whether the proportion of elderly units has an impact on operating expenses, we must isolate the effect of presence of elderly units from the distinct effect of having smaller units that happen to house elderly. Therefore, we include the variable Average Bedrooms Per Unit in the regression in order to draw an accurate coefficient on the variable Percent Elderly. The MassHousing data indicates that there is no relationship between expenses

[8] These example variables are used solely to facilitate understanding of how the relationship between operating expenses and a given independent variable may depend on other project characteristics. No analysis was undertaken that shows the existence or lack thereof of distinctive operating expense associations with number of buildings for projects of different sizes; nor should the use of this example be interpreted as a hypothesis of the author.

per unit and percentage of elderly units, but if the variable Average Bedrooms Per Unit were to be omitted from the regression, Percent Elderly would have appeared to be associated with lower operating expenses per unit. Excluding Average Bedrooms Per Unit would be an instance of omitted variable bias: omission of an independent variable (Average Bedrooms Per Unit) that is a determinant of the dependent variable (Operating Expenses Per Unit) and has an association with a second independent variable (Percent Elderly) is not included as an independent variable, resulting in distortion of the coefficient on the other second variable (Percent Elderly).

The fact that different associations with expenses are seen for some independent variables may indicate that there are other characteristics not included in the data set that are 1) more common in one agency's portfolio than the other, 2) associated with one or more independent variables, and 3) determinants of operating expenses per unit. There may be characteristics that fit the above three criteria that are not included as independent variables in this analysis, the inclusion of which would improve results.

Future data collection and research will benefit from forming hypotheses regarding subpopulations whose operating expenses may be impacted differently by given variables, and by considering what additional determinants of operating expenses may not have been included in this study. Some possibilities in the latter category are discussed in the following section.

Areas for Future Study

Analysis of the individual lender data sets provides insight into additional variables worthy of inclusion in future research. Each MHP and MassHousing provided unique additional variables in their portfolio data set. Though the distinctive composition of projects in each lender's data set compromises external validity of specific regression findings, these regressions are useful for identifying potentially significant determinants of operating expenses.

MHP Data[9]

Variables preliminary determined to be useful for operating expense prediction included:

Years Since Construction or Rehab – Two types of variables related to project age were tested: age as a continuous variable measured in years from closing date, and a dummy variable distinguishing projects that had been constructed or rehabbed in the last ten years. The dummy variable New or Rehab in Last 10 Years has a statistically significant negative correlation with per unit expenses. Property age in quadratic functional form improves the predictive power of the regression as compared to a linear function but is not statistically significant. However, the fact that a quadratic form better fits the data than linear could imply that the relationship between property age and operating expenses varies with the age of the property. For instance, younger projects

[9] Regressions performed on the MHP data can be found in Appendix I.

may not vary much in operating expense levels, while additional years on older projects may have a more material relationship with expense levels.

A number of additional variables were tested and not found to have a statistical or practical significance in predicting operating expenses when controlling for other variables available within the MHP data set. These include:

- Average Unit Area (Average Bedrooms Per Unit was a more powerful predictor and more highly statistically significant; when Average Bedrooms Per Unit was included, Average Unit Area was not statistically significant or useful in adding to the predictive power of the overall regression.)
- Property Condition (tested both as a continuous variable representing the properties' condition grade, and as a dummy variable distinguishing properties of grade B or better from other properties; neither form was statistically significant.)
- Non-Profit Developer/For-Profit Developer
- Management Quality
- Self-Managed/Third Party Managed
- Vacancy Rate

Though no association with operating expenses could be identified for these variables, these findings should not necessarily discourage further study. In particular, is possible that access to a larger sample size of data could produce statistically significant coefficients for some of the variables where there is no statistically significant relationship found in the MHP data set.

MassHousing Data[10]

Variables preliminary determined to be useful for operating expense prediction include:

Individually/Master-Metered Heat: As would be expected, buildings individually metered for heat appear cheaper to operate than master-metered.

Individually/Master-Metered Electricity: Again, as expected, buildings individually metered for electricity appear cheaper to operate than master-metered.

Scattered Site Status: The data indicates that scattered site properties are associated with a lower level of operating expenses per unit. This finding may be contrary to intuition; it is possible that there are other variables correlated to scattered site status not included in the regression that have an impact on operating expenses and bias this result.

Percent Low and Moderate Income: The division between low and moderate income unit designations in the MassHousing data set allowed for a more detailed look at how the affordability mix relates to predicted operating expenses. While it is difficult to determine the precise impact of each of these variables,[11] the inclusion of these variables

[10] Regressions performed on the MassHousing data set can be found in Appendix J.

[11] This issue is discussed further in Appendix I.

does increase the predictive power of the regression, which indicates that the relationship between affordability mix and operating expenses is worth further study.

Construction Type (Concrete/Masonry, Steel, or Wood frame): Inclusion of dummy variables to distinguish projects by construction type improved the predictive power of the regression but did not yield statistically significant coefficients. These results indicate that further research related to physical product type would be worthwhile.

Additional variables were tested and not found to have a statistical or practical significance in predicting operating expenses when controlling for other variables available within the MassHousing data set include:

- Physical/Management PMR
- Distinctions between 4% and 9% LIHTC projects
- Percent Elderly
- Number of Elevators
- Average Stories Per Building
- Type of Heat

Though no statistically significant association with operating expenses could be identified for these variables, these findings should not necessarily discourage inclusion of these variables in future study. It is possible that the inclusion of additional variables in future analysis could reveal significance of these variables as operating expense determinants.

Conclusion

A number of project characteristics hypothesized to have bearing on operating expense levels are confirmed to be determinants of operating expenses by this analysis. Variables that increase operating expenses per unit include LIHTC financing, location in the City of Boston or Boston Market, as well as higher number of average bedrooms per unit and greater percentage of Project-Based Section 8 units. SRO projects and properties located in Gateway Cities, on the other hand, are cheaper to operate on a per unit basis. Surprisingly, the greater the number of buildings, the less costly properties appear to be to operate. Not all independent variables were found to be determinants of operating expenses. The net impact of number of units was fairly inconclusive, with some operating expense categories increasing with additional units and some decreasing. Percent of affordable units did not have an identifiable association with operating expense levels.

Though confirmation of relationships between given project characteristics and operating expense levels is useful as general guidance for affordable housing practitioners, high variance among property expenses and differences in findings across agency portfolios make predictive precision of the regression models weaker than was hoped for underwriting and benchmarking purposes. Inconsistencies in findings across data sets highlight the complexity of the task of predicting operating expenses due to the multitude of project characteristics that have a determining effect.

Further research would benefit from inclusion of additional variables that may be determinants of operating expenses. Specifically, the MHP and MassHousing individual data sets provide a few key variables for inclusion in further studies: property age and years since rehab; identification of whether heat and electricity are individually or master-metered; scattered site status; and distinctions between percentage of units low and moderate income. In addition to the preliminary finding that construction type improves predictive power of regressions, which indicates that this variable worth future study, there could be other design-related variables that would also be appropriate to include in future analysis.

More generally, the availability of data from two agencies reveals two rich avenues for additional exploration. It would be valuable for future research to consider whether the presence of a characteristic or set of characteristics may have bearing on the effect of another characteristic on operating expense levels. Specifically, characteristics where the MHP and MassHousing portfolios respectively differ in composition may define subpopulations of properties for which operating expenses behave differently when additional characteristics are introduced. The independent variables for which findings varied across data sets indicate likely candidates for variables that may operate differently on distinct subpopulations of properties. Further, this study poses the question of what characteristics may be more common to MHP's portfolio as compared to MassHousing's and may be determinants of operating expenses – particularly any characteristics that might have a correlation with any of the independent variables where findings cross data sets differed. Primary program type as defined and employed in this analysis did not appear to be statistically significant in predicting overall operating expenses per unit, but inclusion of dummy variables for program type did serve the important function of adjusting coefficients of other variables to make the MHP and MassHousing findings more consistent. This indicates that program type is an important variable to include as a control, and that future analysis should consider the effect of more specific categories and characteristics related to project financing.

A final overarching lesson of this project is that access to data from diverse sources is greatly beneficial. The collaboration of two agencies in supporting this analysis with data permits identification of areas where further study is needed and strengthens the external validity of the findings, yielding results that are more valuable for all stakeholders.

Appendix A.1: Data Directory

	COMBINED	MHP	MASSHOUSING
MEASURES OF PROPERTY SCALE/PRODUCT TYPE			
Number of Units	X	X	X
Average Bedrooms Per Unit	X	X	X
Number of Buildings	X	X	X
Number of Elevators			X
Scattered Site			X
Average Square Feet Per Unit		X	(Partial data)
Average Stories Per Building			X
Construction Type			X
PROPERTY QUALITY			
Age		X	
Physical PMR			X
Management PMR			X
Property Condition		X	
Years Since Rehab		X	
FINANCING INFORMATION			
LIHTC	X	X	X
9% LIHTC			X
4% LIHTC			X
% PBS8 Units	X	X	X
% Affordable	X	X	X
% Units Low Income			X
% Units Moderate Income			X
Primary Program Type (Interest-Subsidy, Section 8 Program, SHARP/RDAL, Other)	X	X	X
MANAGEMENT/OWNERSHIP INFORMATION			
Management Quality		X	
Self-Managed/Third-Party Managed		X	
Non-Profit Developer/For-Profit Developer		X	
TARGET TENANT INFORMATION			
% Elderly Units			X
SRO	X	X	X
PERFORMANCE CHARACTERISTICS			
Vacancy Rate		X	
UTILITY INFORMATION			
Heat Individually/Master-Metered			X
Electricity Individually/Master-Metered			X
Type of Heat			X
LOCATION AND MARKET CHARACTERISTICS			
Boston Market Location	X	X	X
City of Boston Location	X	X	X
Gateway City Location	X	X	X
AMENITIES			
Services	X	X	X
Security	X	X	X

Appendix A.2: Variable Definitions and Interpretation

Variable Name	Variable Definition	Regression Interpretation (x = Regression Coefficient)*	Data Set(s) In Which Variable Employed**		
			C	MHP	MHFA
MEASURES OF PROPERTY SCALE/PRODUCT TYPE					
Number of Units	Continuous variable	Each additional unit is associated with an x change in operating expenses per unit.	X	X	X
Log Number of Units	Continuous variable representing the natural logarithm of the number of units	Each percentage increase in number of units is associated with an x/100 change in operating expenses per unit.	X	X	X
20 Units or Less* **	Dummy variable: 1 = 20 Units or Less; 0 = 21 Units or More	Properties with 20 or less units are associated with an x change in operating expenses per unit as compared to properties of over 60 units.	X	X	X
21-60 Units* **	Dummy variable: 1 = 21-60 Units; 0 = Less than 21 or more than 60 units	Properties with 21 to 60 units are associated with an x change in operating expenses per unit as compared to properties of over 60 units.	X	X	X
Average Bedrooms Per Unit -Linear Term	Continuous variable	When only linear term is included: Each additional average number of bedrooms is associated with an x change in operating expenses per unit.	X	X	X
Average Bedrooms Per Unit -Quadratic Term	Continuous variable representing the average number of bedrooms per units squared	When quadratic term is included, the linear and quadratic terms should be interpreted as function together. Mathematical interpretation: At a given average number of bedrooms per unit, the increase in operating expenses per unit as compared to a baseline of zero average bedrooms equals: (Linear term coefficient)(Average number of bedrooms) + (Quadratic term coefficient)(Number of average bedrooms squared).^	X	X	X
Number of Buildings - Linear Term	Continuous variable representing the number of buildings	When only linear term is included: Each additional building is associated with an x change in operating expenses per unit.	X	X	X
Number of Buildings - Quadratic Term	Continuous variable representing the number of buildings squared	When quadratic term is included, the linear and quadratic terms should be interpreted as function together. Mathematical interpretation: At a given number of buildings, the increase in operating expenses per unit as compared to a baseline of zero units equals: (Linear term coefficient)(Number of buildings) + (Quadratic term coefficient)(Number of buildings squared).^	X	X	X
Average Square Feet Per Unit	Continuous variable	Each additional average square foot per building is associated with an x change in operating expenses per unit.		X	
Number of Elevators	Continuous variable	Each elevator is associated with an x change in operating expenses per unit.			X

Variable Name	Variable Definition	Regression Interpretation (x = Regression Coefficient)*	Data Set(s) In Which Variable Employed**		
			C	MHP	MHFA
Scattered Site	Dummy variable; 1 = Scattered site property; 0 = Not scattered site property	Scattered site properties are associated with an x change in operating expenses per unit as compared to non-scattered site properties.			X
Average Stories Per Building	Continuous variable	Each additional average story is associated with an x increase in operating expenses per unit.			X
Concrete* **	Dummy variable: 1 = Construction is concrete frame; 0 = Construction is not concrete frame	As compared to wood framed properties, concrete frame properties are associated with an x change in operating expenses per unit.			X
Steel* **	Dummy variable: 1 = Construction is steel frame; 0 = Construction is not steel frame	As compared to wood framed properties, steel frame properties are associated with an x change in operating expenses per unit.			X
PROPERTY QUALITY					
Property Condition	Continuous variable: 1 = Property earned grade of A from MHP Portfolio staff (highest); 2 = Property earned grade of A-; 3 = Property earned grade of B+…7 = Property earned grade of C (lowest)	Each incremental change in grade (from A to A- or from A- to B+ etc.) is associated with an x change in operating expenses per unit.		X	
Property B or Better	Dummy variable: 1 = Property earned Grade B or better on A - C scale graded by MHP Portfolio staff, 0 = Property earned less than Grade B	Properties of grade B or better are associated with an x change in operating expenses per unit as compared to properties earning less than a B grade.		X	
Age - Linear	Continuous variable representing the number of years since the project was completed	When only linear term is included: Each additional year of project age is associated with an x change in operating expenses per unit.		X	
Age - Quadratic	Continuous variable representing the number of the years since the property was completed squared	When quadratic term is included, the linear and quadratic terms should be interpreted as function together. Mathematical interpretation: At a given age, the increase in operating expenses per unit as compared to a baseline of zero years old equals: (Linear term coefficient)(Age in years) + (Quadratic term coefficient)(Age in years squared). ^		X	
New or Rehab in Last 10 Years	Dummy variable: 1 = Constructed or rehabbed within the last 10 years; 0 = Constructed or last rehabbed more than 10 years ago	Properties developed or rehabbed in the last 10 years are associated with an x change in operating expenses per unit as compared to properties constructed or rehabbed more than 10 years ago.		X	

Variable Name	Variable Definition	Regression Interpretation (x = Regression Coefficient)*	Data Set(s) In Which Variable Employed**		
			C	MHP	MHFA
FINANCING INFORMATION					
LIHTC	Dummy variable: 1 = LIHTC property 0 not a LIHTC property	LIHTC properties are associated with an x change in operating expenses per unit as compared to non-LIHTC properties.	X	X	X
Percent Project-Based Section 8 - Linear	Continuous variable representing the proportion of units that are Project-Based Section 8. Values are expressed in decimal form with minimum value 0 and maximum value 1.	When only linear term is included: Each percentage point increase in the number of units that are Project-Based Section 8 is associated with an x/100 change in operating expenses per unit.	X	X	X
Percent Project-Based Section 8 - Quadratic	Continuous variable representing the proportion of units that are Project-Based Section 8 (as defined above) squared	When quadratic term is included, the linear and quadratic terms should be interpreted as function together. Mathematical interpretation: At a given percentage of units that are Project-Based Section 8, the increase in operating expenses per unit as compared to a baseline of 0% Project-Based Section 8 equals: (Linear term coefficient)(Percent Section 8) + (Quadratic term coefficient)(Percent Section 8 squared).^	X	X	X
Percent Affordable - Linear	Continuous variable representing the proportion of units that are affordable (restricted to low or moderate income households.) Values are expressed in decimal form with minimum value 0 and maximum value 1.	When only linear term is included: Each percentage point increase in the number of units that are affordable is associated with an x/100 change in operating expenses per unit.	X	X	X
Percent Affordable - Quadratic	Continuous variable representing the proportion of units that are affordable (as defined above) squared.	When quadratic term is included, the linear and quadratic terms should be interpreted as function together. Mathematical interpretation: At a given percentage of units that are affordable, the increase in operating expenses per unit as compared to a baseline of 0% affordable equals: (Linear term coefficient)(Percent affordable) + (Quadratic term coefficient)(Percent affordable squared).^	X	X	X
9% LIHTC	Dummy variable: 1 = 9% LIHTC financed; 0 = Not 9% LIHTC financed	9% LIHTC properties are associated with an x change in operating expenses per unit as compared to non-9% LIHTC properties.			X
4% LIHTC	Dummy variable: 1 = 4% LIHTC financed; 0 = Not 4% LIHTC financed	4% LIHTC properties are associated with an x change in operating expenses per unit as compared to non-4% LIHTC properties.			X

Variable Name	Variable Definition	Regression Interpretation (x = Regression Coefficient)*	Data Set(s) In Which Variable Employed**		
			C	MHP	MHFA
Percent Low Income - Linear	Continuous variable representing the proportion of units that are low income, defined as below 50% AMI	When only linear term is included: Each percentage point increase in the number of units that are low income is associated with an x/100 change in operating expenses per unit.			X
Percent Low Income - Quadratic	Continuous variable representing the proportion of units that are low income (as defined above) squared	When quadratic term is included, the linear and quadratic terms should be interpreted as function together. Mathematical interpretation: At a given percentage of units that are low income the increase in operating expenses per unit as compared to a baseline of 0% affordable equals: (Linear term coefficient)(Percent low income + (Quadratic term coefficient)(Percent low income squared).			X
Percent Moderate Income - Linear	Continuous variable representing the proportion of units that are low income, defined as below 80% AMI, squared	When only linear term is included: Each percentage point increase in the number of units that are moderate income is associated with an x/100 change in operating expenses per unit.			X
Percent Moderate Income - Quadratic	Continuous variable representing the proportion of units that are low income (as defined above) squared	When quadratic term is included, the linear and quadratic terms should be interpreted as function together. Mathematical interpretation: At a given percentage of units that are moderate income the increase in operating expenses per unit as compared to a baseline of 0% affordable equals: (Linear term coefficient)(Percent moderate income + (Quadratic term coefficient)(Percent moderate income squared).^			X
Interest Subsidy Program* **	Dummy variable: 1 = Principal project financing is Section 236 and/or Section 13A, and project does not have other subsidy (excluding tenant-based subsidies) for a greater number of units than are covered by interest reduction subsidies; 0 = Principal financing is not Section 236 or Section 13A	Properties where the principal project financing is an Interest Subsidy program are associated with an x increase in operating expenses per unit as compared to projects where the primary financing program is not an interest subsidy program, Section 8 program, or SHARP/RDAL.	X	X	X
Section 8 Program* **	Dummy variable: 1 = Principal project financing is a Project-Based Section 8 assistance contract or Section 23, and project does not have other subsidy for a greater number of units than are covered by the Section 8 contract; 0 = Principal project financing is not a Project-Based Section 8 Program	Properties where the principal project financing is the Section 8 Program are associated with an x increase in operating expenses per unit as compared to projects where the primary financing program is not an Interest Subsidy program, Section 8 program, or SHARP/RDAL.	X	X	X
SHARP/RDAL* **	Dummy variable: 1 = Principal project financing is SHARP and /or RDAL; 0 = Principal project financing is not SHARP and/or RDAL	Properties where the principal project financing is SHARP or RDAL are associated with an x increase in operating expenses per unit as compared to projects where the primary financing program is not an Interest Subsidy program, Section 8 program, or SHARP/RDAL.	X	X	X

Variable Name	Variable Definition	Regression Interpretation (x = Regression Coefficient)*	Data Set(s) In Which Variable Employed**		
			C	MHP	MHFA
MANAGEMENT/OWNERSHIP INFORMATION					
Management Quality	Continuous variable representing the grade received from MHP Portfolio staff for management quality (1 is highest, 6 is lowest)	Each incremental change in management quality (from 1 to 2, from 2 to 3 etc.) is associated with an x change in operating expenses per unit.		X	
Self-Managed	Dummy variable; 1 = Self-managed; 0 = Third-party managed	Properties that are self-managed are associated with an x change in operating expenses per unit as compared to third-party managed properties.		X	
Non-Profit Developer	Dummy variable: 1 = Project developed/rehabbed and controlled by a non-profit developer; 0 = Project developed/rehabbed and controlled by a for-profit developer	Properties developed by a non-profit are associated with an x change in operating expenses per unit as compared to properties developed by a for-profit.		X	
TARGET TENANT INFORMATION					
SRO	Dummy variable: 1 = SRO property, 0 = Not SRO property	SRO properties are associated with an x change in operating expenses per unit as compared to non-SRO properties.	X	X	X
Percent Elderly	Continuous variable representing the proportion of units that are elderly	Each percentage point increase in the number of units that are elderly is associated with an x/100 change in operating expenses per unit.			X
PERFORMANCE CHARACTERISTICS					
Vacancy	Continuous variable representing percent of units that are vacant	Each percentage point increase in the number of units that are vacant is associated with an x change in operating expenses per unit.		X	
UTILITY INFORMATION					
Heat Individually Metered	Dummy variable: 1 = Heat individually metered; 0 = Heat master-metered	Properties individually metered for heat are associated with an x change in operating expenses per unit as compared to other properties.			X
Electricity Individually Metered	Dummy variable: 1 = Electricity individually metered; 0 = Electricity master-metered	Properties individually metered for electricity are associated with an x change in operating expenses per unit as compared to other properties.			X
Gas Heat* **	Dummy variable: 1 = Property has gas heat; 0 = Property does not have gas heat	As compared to oil heated properties, properties with gas heat are associated with an x change in operating expenses per unit.			X
Electric Heat* **	Dummy variable: 1 = Property has electric heat; 0 = Property does not have electric heat	As compared to oil heated properties, properties with electric heat are associated with an x change in operating expenses per unit.			X

Variable Name	Variable Definition	Regression Interpretation (x = Regression Coefficient)*	Data Set(s) In Which Variable Employed**		
			C	MHP	MHFA
LOCATION AND MARKET CHARACTERISTICS					
Boston Market Location	Dummy variable: 1 = Located in Boston, Brookline, Cambridge, or Somerville; 0 = Not located in one of these municipalities	Properties located in the Boston Market are associated with an x change in operating expenses per unit as compared to properties in other locations.	X	X	X
City of Boston Location	Dummy variable: 1 = Located in City of Boston, 0 = Not located in the City of Boston	Properties located in the City of Boston are associated with an x change in operating expenses per unit as compared to properties in Brookline, Cambridge and Somerville. (The increase associated with City of Boston location as compared to non-Boston market locations is found by summing the coefficient on City of Boston Location and the coefficient on Boston Market Location.)	X	X	X
Gateway City Location	Dummy variable: 1 = Located in Barnstable, Brockton, Chelsea, Chicopee, Everett, Fall River, Fitchburg, Haverhill, Holyoke, Lawrence, Leominster, Lowell, Lynn, Malden, Methuen, New Bedford, Pittsfield, Quincy, Revere, Salem, Springfield, Taunton, Westfield, or Worcester 0 = Not located in one of these municipalities	Properties located in Gateway Cities are associated with and x change in operating expense levels as compared to properties in other locations.	X	X	X
AMENITIES					
Security	Dummy variable: 1 = Some level of security is provided; 0 = No security is provided. Source: costs reflected in operating expenses	Properties that provide some level of security are associated with an x change in operating expenses per unit as compared to properties with no security expense.	X	X	X
Services	Dummy variable: 1 = Some level of services are provided; 0 = No services are provided. Source: costs reflected in operating expenses	Properties that provide some level of services are associated with an x change in operating expenses per unit as compared to projects with no services expense.	X	X	X

* All coefficients should be interpreted as the association when all other variables in the regression are included as controls.

** C = Combined Data Set; MHFA = MassHousing

*** Note that in order to compare the effect of a variable that falls into more than two categories, regression analysis employs dummy variables for all but one category. The excluded category becomes the baseline project scenario to which the effect of the other categories are compared.

^ Appendix C provides calculation interpretation of quadratic functional forms for all variables after the regression tables. Appendix E also provides calculations for select quadratic functional forms after the regression tables.

Appendix B: Quadratic and Logarithmic Regression Function Interpretation

Quadratic and logarithmic regressions are employed in this analysis when the change in the dependent variable associated with an additional unit of the independent variable varies with the magnitude of the independent variable.

Quadratics functions:
Several independent variables are modeled with both a linear term and a quadratic term. In these instances, mathematical interpretation is as follows:

- The increase/decrease in predicted operating expenses per unit at a given value of the independent variable can be compared to the baseline representing a project for which the independent variable equals zero. The equation is: (Linear term coefficient)(Independent variable value) + (Quadratic coefficient)(Independent variable value squared.)
- Change associated with an additional unit increase of the independent variable at a given value of the independent variable is the derivate of the function: (Linear term coefficient) + 2 (Quadratic term coefficient)(Independent variable value)

The definitions provided in Appendix A for each variable serve to facilitate interpretation of quadratic functions. Specific quantitative interpretation for the quadratic functions in the combined data set analysis is included after the regressions in Appendix C and select regressions in Appendix F.

Logarithmic functions:
Use of logarithmic functions in this analysis is limited.

Level-log form involves use of a log form for one or more independent variables. Interpretation in this instance is: A one percent increase in the independent variable is associated a (coefficient)(.01) change in the dependent variable. The only variable for which level-log form was found to be a good fit was Number of Units. Definition for interpretation of Log Number of Units is provided in Appendix A.

Log-level form involves use of a log of the dependent variable. In this circumstance, interpretation of the coefficients is: A one unit increase in the independent variable is associated with a 100(coefficient) percent change in the dependent variable. Log-level form was not found to enhance the analysis, and this form is provided along with other regression forms in Appendices E, I, and J only as reference.

Appendix C: Comparison of Regressions Across Data Sets

This regression table shows the best predictive regression form for combined data next to the same regressions performed on the individual agency data sets.

The best predictive regression form was achieved by including all variables in the combined data set and employing select quadratic functional form for some independent variables. Quadratic terms for independent variables are included when their coefficient is jointly significant[1] with the linear term coefficient and inclusion improves the predictive power of the regression. Number of Units was determined to have greater predictive power when modeled using dummy variables for unit range categories. Several variations of unit range categories were tested, and the categories 0-20 units, 21–60 units, and 61 or more units provided the best predictive model.

[1] The 10% significance level is used.

	Combined		MHP		MassHousing	
	(1)	(2)	(3)	(4)	(5)	(6)
	2009 Op. Ex./ Unit	2008 Op. Ex./ Unit	2009 Op. Ex./ Unit	2008 Op. Ex./ Unit	2009 Op. Ex./ Unit	2008 Op. Ex./ Unit
20 Units or Less	-441.4*	-390.1	-69.88	707.5	826.8**	504.4
	(263.3)	(275.4)	(476.1)	(599.6)	(341.4)	(416.3)
21 - 60 Units	88.11	154.8	235.4	354.8	245.8	401.2**
	(158.1)	(168.1)	(334.5)	(413.3)	(178.5)	(193.0)
Average Bedrooms Per Unit - Linear Term	123.4	434.3	-280.9	33.48	1,101*	1,260**
	(591.1)	(551.4)	(1,101)	(1,220)	(611.0)	(624.9)
Average Bedrooms Per Unit - Quadratic Term	306.3*	215.1	342.7	256.9	10.30	-28.22
	(180.3)	(162.3)	(323.4)	(351.3)	(184.6)	(184.3)
Number of Buildings - Linear Term	-43.95**	-43.80**	-24.34	-3.761	-40.21**	-45.12**
	(17.27)	(17.24)	(54.53)	(59.22)	(18.21)	(18.16)
Number of Buildings - Quadratic Term	0.635**	0.691**	-0.0758	0.0207	0.566*	0.668**
	(0.292)	(0.274)	(1.522)	(1.555)	(0.291)	(0.277)
LIHTC	165.6	268.5*	1,139***	1,872***	-7.269	21.27
	(132.2)	(141.1)	(304.5)	(407.7)	(140.4)	(141.7)
Percent PBS8 - Linear Term	-367.5	-1,184	1,502	755.9	-1,205	-1,585*
	(845.7)	(823.8)	(1,578)	(1,795)	(1,036)	(912.8)
Percent PBS8 - Quadratic Term	1,624*	2,641***	-480.3	658.9	2,699**	3,127***
	(916.8)	(863.1)	(1,691)	(1,915)	(1,105)	(948.7)
Percent Affordable	-331.8	-328.2	499.9	377.4	-1,561***	-1,427***
	(262.7)	(287.4)	(374.7)	(389.4)	(371.8)	(417.1)
Interest Subsidy	177.8	488.4**	-712.4	64.61	433.0*	461.2
	(192.4)	(209.9)	(915.5)	(1,113)	(248.5)	(306.2)
Section 8 Program	219.2	360.1	0	0	313.3	282.9
	(357.5)	(322.4)	(0)	(0)	(458.1)	(441.8)
SHARP/RDAL	112.0	284.3	0	0	-255.0	-299.6
	(202.0)	(221.9)	(0)	(0)	(220.0)	(256.9)
SRO	-2,118***	-2,153***	-1,893***	-1,566**	0	0
	(408.1)	(436.1)	(553.9)	(712.1)	(0)	(0)
Boston Market Location	1,407***	1,578***	1,535***	1,790***	1,526***	1,784***
	(307.1)	(340.2)	(342.9)	(365.7)	(423.1)	(473.5)
City of Boston Location	585.7*	471.0	-314.7	-245.7	426.8	235.2
	(305.1)	(344.8)	(269.8)	(323.2)	(418.9)	(473.8)
Gateway City Location	-259.5*	-284.8*	-296.2	-314.9	-262.5*	-256.9
	(149.2)	(153.3)	(314.1)	(312.8)	(158.9)	(168.2)
Services	341.3***	226.0*	958.1***	688.0*	284.9**	205.9
	(128.5)	(134.3)	(328.9)	(365.3)	(136.4)	(142.0)
Security	45.86	35.78	367.2	46.38	763.4***	754.5***
	(28.11)	(26.60)	(313.7)	(294.3)	(136.4)	(141.4)
Constant	5,657***	5,385***	4,783***	3,748***	5,415***	5,414***
	(513.3)	(516.2)	(954.7)	(1,189)	(542.9)	(564.0)
Observations	625	602	168	157	451	439
R-squared	0.502	0.501	0.603	0.609	0.508	0.500

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Quadratic Functions Interpretation

Independent Variable: Average Bedrooms Per Unit
Dependent Variable: Operating Expenses Per Unit

Regressions (1) and (2) - Combined Data Set

As compared to properties with zero bedrooms per unit, a property that has this number of average bedrooms per unit:			
1	2	3	
is estimated to have this increased/decreased level of operating expenses per unit per 2009 data:			
$430	$1,472	$3,127	
and this estimated increased/decreased level of operating expenses per unit per 2008 data:			
$649	$1,729	$3,239	

Regressions (3) and (4) - MHP Data Set

As compared to properties with zero bedrooms per unit, a property that has this number of average bedrooms per tunit:			
1	2	3	
is estimated to have this increased/decreased level of operating expenses per unit per 2009 data:			
$63	$811	$2,244	
and this estimated increased/decreased level of operating expenses per unit per 2008 data:			
$290	$1,095	$2,413	

Regressions (5) and (6) - MassHousing Data Set

As compared to properties with zero bedrooms per unit, a property that has this number of average bedrooms per unit:			
1	2	3	
is estimated to have this increased/decreased level of operating expenses per unit per 2009 data:			
$1,111	$2,243	$3,396	
and this estimated increased/decreased level of operating expenses per unit per 2008 data:			
$1,232	$2,407	$3,526	

Independent Variable: Number of Buildings
Dependent Variable: Operating Expenses Per Unit

Regressions (1) and (2) - Combined Data Set

As compared to a baseline of a property with zero buildings, this number of buildings:									
1	2	3	4	5	6	7	8	9	10
is estimated to have this increased/decreased level of operating expenses per unit per 2009 data:									
-$43	-$85	-$126	-$166	-$204	-$241	-$277	-$311	-$344	-$376
and this estimated increased/decreased level of operating expenses per unit per 2008 data:									
-$43	-$85	-$125	-$164	-$202	-$238	-$273	-$306	-$338	-$369

Regressions (3) and (4) - MHP Data Set

As compared to a baseline of a property with zero buildings, this number of buildings:									
1	2	3	4	5	6	7	8	9	10
is estimated to have this increased/decreased level of operating expenses per unit per 2009 data:									
-$24	-$49	-$74	-$99	-$124	-$149	-$174	-$200	-$226	-$251
and this estimated increased/decreased level of operating expenses per unit per 2008 data:									
-$4	-$7	-$11	-$15	-$18	-$22	-$25	-$29	-$32	-$36

Regressions (5) and (6) - MassHousing Data Set

As compared to a baseline of a property with zero buildings, this number of buildings:									
1	2	3	4	5	6	7	8	9	10
is estimated to have this increased/decreased level of operating expenses per unit per 2009 data:									
-$40	-$78	-$116	-$152	-$187	-$221	-$254	-$285	-$316	-$346
and this estimated increased/decreased level of operating expenses per unit per 2008 data:									
-$44	-$88	-$129	-$170	-$209	-$247	-$283	-$318	-$352	-$384

Independent Variable: Percent Project-Based Section 8
Dependent Variable: Operating Expenses Per Unit

Regressions (1) and (2) - Combined Data Set

As compared to a baseline of a property with no Project-Based Section 8 units, this percentage of Project-Based Section 8 Units:									
10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
is estimated to have this increased/decreased level of operating expenses per unit per 2009 data:									
-$21	-$9	$36	$113	$222	$364	$539	$745	$985	$1,257
and this estimated increased/decreased level of operating expenses per unit per 2008 data:									
-$92	-$131	-$118	-$51	$68	$240	$465	$743	$1,074	$1,457

Regressions (3) and (4) - MHP Data Set

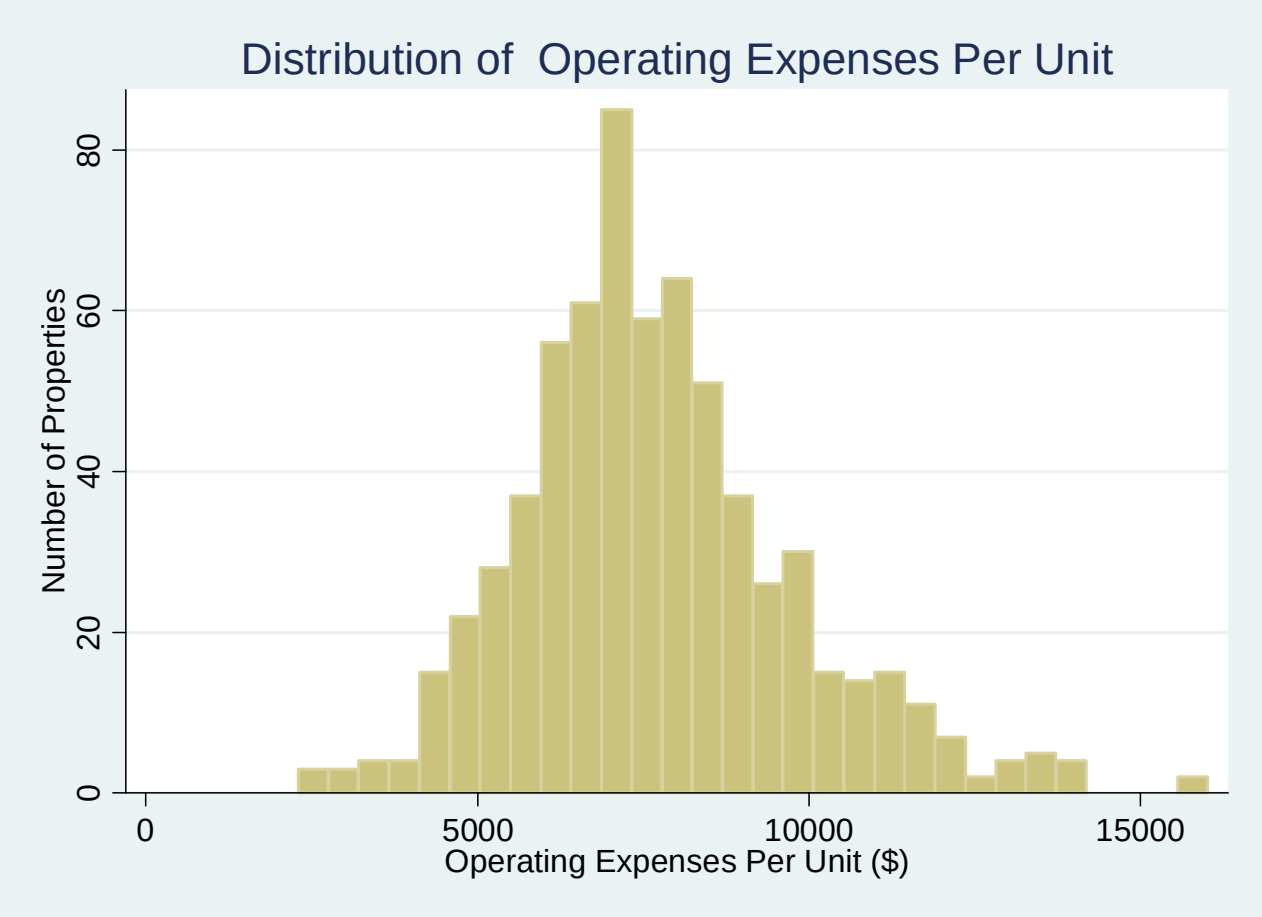
As compared to a baseline of a property with no Project-Based Section 8 units, this percentage of Project-Based Section 8 Units:									
10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
is estimated to have this increased/decreased level of operating expenses per unit per 2009 data:									
$145	$281	$407	$524	$631	$728	$816	$894	$963	$1,022
and this estimated increased/decreased level of operating expenses per unit per 2008 data:									
$82	$178	$286	$408	$543	$691	$852	$1,026	$1,214	$1,415

Regressions (5) and (6) - MassHousing Data Set

As compared to a baseline of a property with no Project-Based Section 8 units, this percentage of Project-Based Section 8 Units:									
10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
is estimated to have this increased/decreased level of operating expenses per unit per 2009 data:									
-$94	-$133	-$119	-$50	$72	$249	$479	$763	$1,102	$1,494
and this estimated increased/decreased level of operating expenses per unit per 2008 data:									
-$127	-$192	-$194	-$134	-$11	$175	$423	$733	$1,106	$1,542

Exhibit D.1: Histogram of Operating Expenses Per Unit Distribution

The below histogram shows how the properties in the combined data set (2009) are distributed in terms of their operating expenses per unit.



Operating Expenses Per Unit (2009)

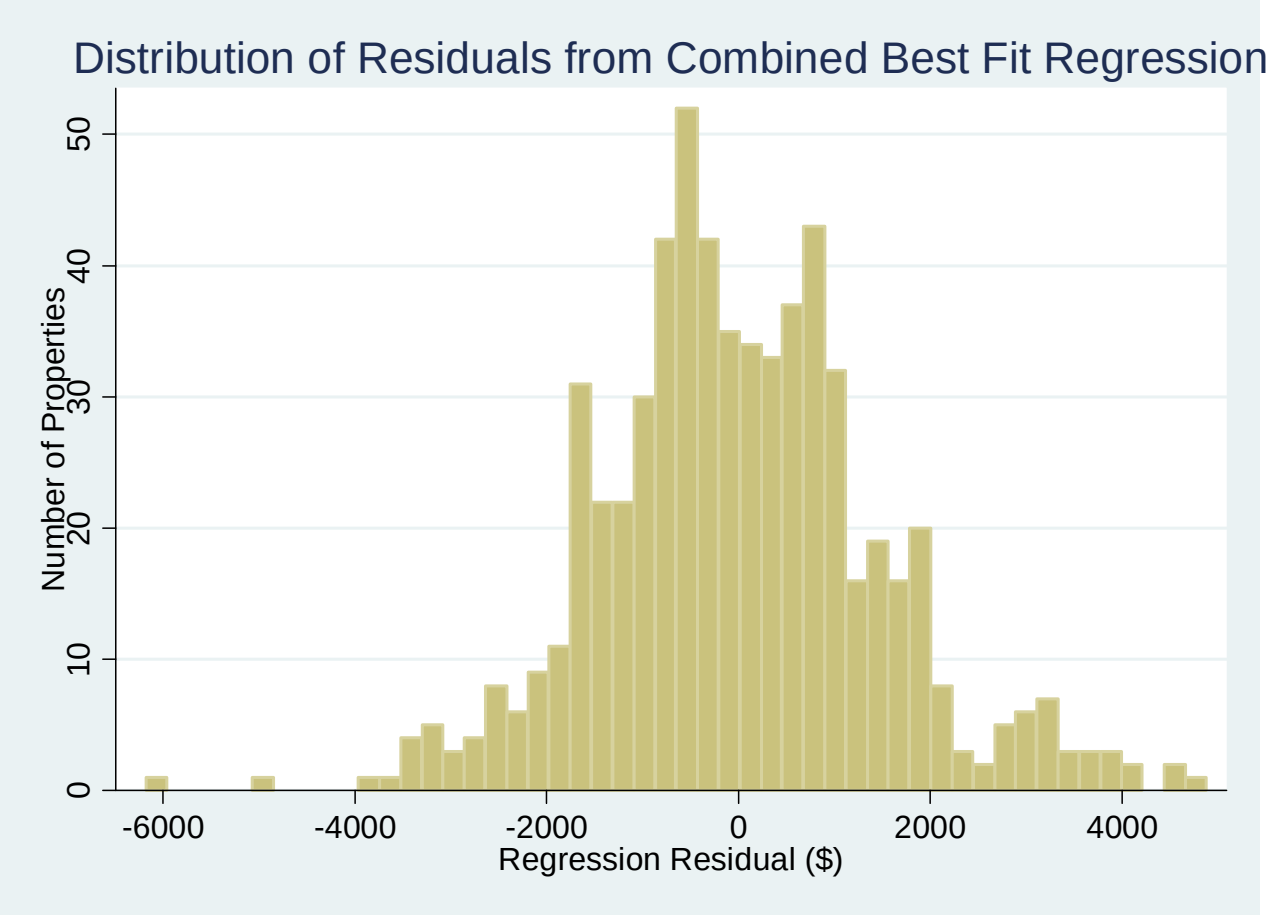
Minimum:	$2,373
Maximum:	$16,014
Mean:	$7,694
Standard Deviation:	$2,052

Exhibit D. 2: Histogram of Residuals Distribution

The following histogram provides a visual representation of how well the combined data set best predictive fit regression for the 2009 data (Exhibit C) predicts actual operating expenses for properties in the combined data set. The residual of a regression is defined as the difference between the actual value of the independent variable (in this case, Operating Expenses Per Unit) and the value that the regression predicts for the independent variable.

The below shows the distribution of the residuals for all properties in the combined data set. Negative residuals mean that the actual is less than the prediction by the magnitude of the residual; positive residuals mean that the actual exceeds the prediction by the magnitude of the residual. 50% of the data has a residual of an absolute value of less than $896.[1] As can be seen below, the vast majority of the data has a residual between $-2,000 and $2,000, though there are tails in both the positive and negative directions outside this range. The R^2 value of .502 for this regression implies that the model explains about half of the variation in operating expenses between properties; the other half of the variation is reflected in the residuals.

[1] 25th percentile and 75th percentiles of the data are $ -896 and $868 respectively.



Residuals of Best Predictive Fit Regression (Appendix C, 2009) [2]

Minimum: $-6,176
Maximum: $4,883
Standard Deviation: $1,469

[2] By design, regressions always have an average residual of zero.

Appendix E: Combined Data Set Regressions

This regression table shows several versions of regressions performed to analyze the relationship between the independent variables and operating expenses per unit. Linear regressions both including and excluding the variables Services and Security (variables understood by definition to be associated with the dependent variable, but the inclusion of which was hoped to be useful for underwriting purposes) are provided. The evolution of how number of units was modeled is also charted. While logarithmic form was found to be a better fit than standard linear form for the variable Number of Units, the unit range dummy variables distinguishing projects 0–20 units, 21–60 units, and 61 or more units provided the best model.

A few variations of the dependent variable were also tested to determine best predictive model. Log-level regression form, in which the dependent variables are expressed in logarithmic form, was tested. Standard level-level regression was determined to be a slightly better fit, but the log-level regression for overall operating expenses per unit is included as reference. Operating expenses per unit net of utilities was also tested as a dependent variable to assess whether isolating non-utility expenses would make operating expenses easier to predict; as the fit was not as good as for operating expenses per unit including utilities, this form is not included in the report.

Regression descriptions:

- (1) and (7): Linear regression of variables in combined data set only; excludes Services and Security
- (2) and (8): Linear regression of variables in combined data set only; includes Services and Security
- (3) and (9): Same as above but Log Number of Units substituted for Number of Units
- (4) and (10): Same as above but unit range dummy variables substituted for Number of Units
- (5) and (11): Includes quadratic terms and unit range dummy variables (Same as Regressions (1) and (2) of Appendix C)
- (6) and (12): Same as above but Log Operating Expenses is the dependent variable

2009 Data

	(1) Op. Ex/Unit	(2) Op. Ex/Unit	(3) Op. Ex/Unit	(4) Op. Ex/Unit	(5) Op. Ex/Unit	(6) Log Op. Ex/Unit
Number of Units	0.715	0.310				
	(0.892)	(0.949)				
Log Number of Units			-63.51			
			(96.02)			
20 Units or Less				-232.3	-441.4*	0.0759*
				(255.6)	(263.3)	(0.0394)
21 - 60 Units				154.6	88.11	0.0144
				(155.0)	(158.1)	(0.0235)
Average Bedroms Per Unit - Linear Term	1,080***	1,109***	1,064***	1,130***	123.4	0.157**
	(164.6)	(165.1)	(160.6)	(165.7)	(591.1)	(0.0715)
Average Bedroms Per Unit - Quadratic Term					306.3*	0.000889
					(180.3)	(0.0191)
Number of Buildings - Linear Term	-17.59*	-19.67**	-14.95	-18.66*	-43.95**	-0.00592**
	(9.859)	(9.921)	(9.540)	(9.522)	(17.27)	(0.00240)
Number of Buildings - Quadratic Term					0.635**	8.79e-05**
					(0.292)	(3.95e-05)
LIHTC	243.8*	198.6	218.5	139.3	165.6	-0.0120
	(140.4)	(139.7)	(134.8)	(135.8)	(132.2)	(0.0185)
Percent PBS8 - Linear Term	1,145***	1,119***	1,115***	1,125***	-367.5	-0.0860
	(345.4)	(340.8)	(341.7)	(339.2)	(845.7)	(0.139)
Percent PBS8 - Quadratic Term					1,624*	0.242*
					(916.8)	(0.140)
Percent Affordable	-324.3	-325.7	-373.9	-296.2	-331.8	-0.162***
	(260.6)	(259.1)	(266.2)	(261.3)	(262.7)	(0.0487)
Interest Subsidy	295.7	201.3	255.6	186.0	177.8	0.0160
	(198.4)	(199.1)	(201.7)	(197.0)	(192.4)	(0.0319)
Section 8 Program	497.6	398.9	460.9	373.1	219.2	0.0398
	(357.6)	(358.2)	(365.3)	(362.0)	(357.5)	(0.0535)
SHARP/RDAL	139.7	137.0	191.6	126.3	112.0	-0.0327
	(196.1)	(193.7)	(195.3)	(199.7)	(202.0)	(0.0311)
SRO	-1,961***	-1,921***	-1,983***	-1,840***	-2,118***	0
	(373.2)	(379.5)	(384.7)	(395.7)	(408.1)	(0)
Boston Market Location	1,478***	1,423***	1,436***	1,424***	1,407***	0.190***
	(297.8)	(303.5)	(304.2)	(307.8)	(307.1)	(0.0442)
City of Boston Location	628.5**	674.1**	655.9**	660.6**	585.7*	0.0616
	(302.8)	(306.8)	(306.5)	(311.7)	(305.1)	(0.0434)
Gateway City Location	-181.6	-193.3	-178.0	-182.0	-259.5*	-0.0189
	(145.6)	(144.6)	(143.6)	(142.1)	(149.2)	(0.0223)
Services		377.1***	407.9***	361.1***	341.3***	0.0308*
		(125.1)	(125.6)	(126.9)	(128.5)	(0.0182)
Security		35.57	42.13	38.06	45.86	0.00795**
		(28.49)	(29.14)	(27.92)	(28.11)	(0.00362)
Constant	4,823***	4,696***	5,012***	4,693***	5,657***	8.664***
	(337.0)	(343.5)	(552.6)	(316.7)	(513.3)	(0.0697)
Observations	625	625	625	625	625	457
R-squared	0.481	0.488	0.489	0.491	0.502	0.478

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

2008 Data

	(7) Op. Ex/Unit	(8) Op. Ex/Unit	(9) Op. Ex/Unit	(10) Op. Ex/Unit	(11) Op. Ex/Unit	(12) Log Op. Ex/Unit
Number of Units	0.130	-0.178				
	(0.761)	(0.794)				
Log Number of Units			-72.93			
			(97.38)			
20 Units or Less				-199.1	-390.1	-0.0615
				(267.3)	(275.4)	(0.0422)
21 - 60 Units				226.4	154.8	0.0299
				(167.0)	(168.1)	(0.0244)
Average Bedroms Per Unit - Linear Term	1,052***	1,073***	1,040***	1,104***	434.3	0.120*
	(152.5)	(152.8)	(153.4)	(155.1)	(551.4)	(0.0688)
Average Bedroms Per Unit - Quadratic Term					215.1	0.00955
					(162.3)	(0.0192)
Number of Buildings - Linear Term	-13.45	-14.93	-12.64	-16.14*	-43.80**	-0.00618***
	(9.542)	(9.600)	(9.416)	(9.101)	(17.24)	(0.00227)
Number of Buildings - Quadratic Term					0.691**	9.31e-05***
					(0.274)	(3.55e-05)
LIHTC	333.5**	303.5**	325.0**	239.0*	268.5*	0.0509**
	(141.6)	(142.9)	(141.9)	(143.4)	(141.1)	(0.0232)
Percent PBS8 - Linear Term	1,247***	1,226***	1,230***	1,221***	-1,184	-0.0530
	(309.9)	(307.1)	(308.0)	(306.2)	(823.8)	(0.111)
Percent PBS8 - Quadratic Term					2,641***	0.239**
					(863.1)	(0.114)
Percent Affordable	-325.3	-316.6	-352.1	-264.1	-328.2	-0.0291
	(283.7)	(282.5)	(286.9)	(285.8)	(287.4)	(0.0453)
Interest Subsidy	570.6***	496.6**	540.0**	466.5**	488.4**	0.0855***
	(206.6)	(210.0)	(218.2)	(214.3)	(209.9)	(0.0329)
Section 8 Program	679.3**	602.4*	647.5*	571.8*	360.1	0.0677
	(320.4)	(323.7)	(331.9)	(330.2)	(322.4)	(0.0443)
SHARP/RDAL	315.1	313.5	363.0*	303.6	284.3	0.0638
	(209.4)	(207.9)	(215.5)	(221.0)	(221.9)	(0.0391)
SRO	-2,065***	-2,044***	-2,089***	-1,933***	-2,153***	-0.318***
	(391.0)	(395.7)	(398.8)	(415.0)	(436.1)	(0.0765)
Boston Market Location	1,601***	1,564***	1,571***	1,550***	1,578***	0.198***
	(339.8)	(341.8)	(342.8)	(345.2)	(340.2)	(0.0412)
City of Boston Location	546.6	580.8*	567.3	574.5	471.0	0.0315
	(348.1)	(349.4)	(350.8)	(354.4)	(344.8)	(0.0469)
Gateway City Location	-200.3	-204.2	-199.1	-206.7	-284.8*	-0.0388*
	(149.3)	(149.1)	(148.3)	(146.1)	(153.3)	(0.0214)
Services		282.6**	307.1**	263.1*	226.0*	0.0226
		(134.9)	(133.5)	(134.3)	(134.3)	(0.0203)
Security		22.21	26.32	23.79	35.78	0.00503
		(28.01)	(28.28)	(26.52)	(26.60)	(0.00350)
Constant	4,843***	4,741***	5,046***	4,668***	5,385***	8.546***
	(340.0)	(343.9)	(545.5)	(336.4)	(516.2)	(0.0797)
Observations	602	602	602	602	602	602
R-squared	0.482	0.486	0.487	0.489	0.501	0.427

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Appendix F: Operating Expense Categories Regressions

Each operating expense category is used as a dependent variable in the attached regression table. Two regression models are included for each dependent variable:

- Odd numbered regressions employ exclusively linear terms for all independent variables. This form is included to facilitate interpretation.
- Even numbered regressions employ quadratic terms for some variables, consistent with the best predictive fit model used for overall operating expenses in Appendix C and Regressions (5) and (11) in Appendix D.

2009 Data

	(1) Admin./ Mgmt. Fee/Unit	(2) Admin./ Mgmt. Fee/Unit	(3) Maintenance/ Unit	(4) Maintenance/ Unit	(5) Utilities Net Water/Unit	(6) Utilities Net Water/Unit	(7) Water/Unit	(8) Water/Unit
20 Units or Less	-250.4**	-287.7***	194.8	64.44	-149.3	-170.0*	-50.14	-70.14
	(105.2)	(110.2)	(151.5)	(157.3)	(93.21)	(97.75)	(44.90)	(47.51)
21 - 60 Units	98.65	88.07	188.2**	148.0*	-64.31	-72.08	-25.02	-30.84
	(61.93)	(62.69)	(84.97)	(87.74)	(53.47)	(54.17)	(25.45)	(25.23)
Average Bedrooms Per Unit - Linear Term	108.8*	-227.0	586.1***	-385.0	48.50	134.9	253.3***	224.3***
	(55.49)	(151.6)	(107.5)	(390.1)	(54.29)	(202.2)	(21.76)	(68.41)
Average Bedrooms Per Unit - Quadratic Term		94.92**		280.1**		-17.67		11.65
		(42.59)		(123.1)		(60.74)		(19.65)
Number of Buildings - Linear Term	-0.673	0.855	-4.229	-6.754	-4.319	-14.53*	-1.938	-7.405***
	(2.855)	(5.845)	(4.764)	(9.783)	(3.993)	(7.674)	(1.363)	(2.632)
Number of Buildings - Quadratic Term		-0.0275		0.0881		0.241		0.130***
		(0.0892)		(0.153)		(0.180)		(0.0447)
LIHTC	153.0***	155.4***	-54.18	-41.03	109.0**	114.6**	1.220	4.111
	(57.71)	(58.21)	(75.58)	(71.25)	(49.96)	(49.82)	(22.87)	(22.70)
Percent PBS8 - Linear Term	418.6***	227.4	368.2**	-547.9	157.1*	19.38	-29.66	-40.86
	(116.8)	(414.2)	(184.1)	(460.2)	(93.91)	(337.8)	(43.85)	(170.2)
Percent PBS8 - Quadratic Term		204.7		992.3*		153.0		11.45
		(417.1)		(512.0)		(354.1)		(175.0)
Percent Affordable	-27.08	-35.53	30.90	0.723	-56.87	-54.54	2.833	5.024
	(122.6)	(122.6)	(132.4)	(131.0)	(100.3)	(100.8)	(46.58)	(46.44)
Interest Subsidy	-151.4*	-160.3*	-63.30	-82.92	587.0***	592.7***	107.6***	107.0***
	(86.06)	(86.27)	(105.8)	(102.9)	(78.47)	(79.25)	(39.97)	(40.29)
Section 8 Program	-151.5	-180.8	159.5	46.08	232.9**	225.5**	135.5***	133.3***
	(117.2)	(114.6)	(220.1)	(206.6)	(104.2)	(107.2)	(49.92)	(51.38)
SHARP/RDAL	-263.7***	-265.3***	225.4**	218.7**	343.5***	341.4***	129.9***	128.5***
	(78.33)	(78.98)	(105.5)	(108.3)	(80.41)	(79.40)	(39.42)	(39.06)
SRO	-533.3***	-638.6***	-526.2**	-823.8***	-330.9**	-290.5*	-39.88	-40.22
	(162.7)	(164.2)	(210.9)	(211.7)	(147.3)	(154.6)	(60.01)	(61.84)
Boston Market Location	364.8***	363.3***	402.3**	400.8**	341.8***	334.1***	212.6***	203.7***
	(127.3)	(129.0)	(168.0)	(180.8)	(92.86)	(92.92)	(42.43)	(42.77)
City of Boston Location	24.56	9.545	53.06	-2.611	189.4**	188.0**	56.62	56.39
	(128.6)	(129.6)	(170.4)	(177.5)	(92.33)	(91.22)	(42.75)	(42.78)
Gateway City Location	-62.12	-68.24	-154.1**	-184.5**	106.5**	89.34*	10.50	-0.980
	(60.55)	(62.05)	(71.21)	(76.76)	(51.59)	(52.69)	(26.65)	(26.75)
Constant	1,726***	2,012***	1,028***	1,880***	622.4***	579.7***	6.774	47.01
	(116.5)	(159.4)	(194.4)	(328.7)	(115.2)	(182.0)	(48.16)	(65.42)
Observations	625	625	625	625	625	625	625	625
R-squared	0.232	0.238	0.301	0.329	0.286	0.290	0.395	0.400

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

	(9) Services/Unit	(10) Services/Unit	(11) Security/Unit	(12) Security/Unit	(13) Taxes/Unit	(14) Taxes/Unit	(15) Insurance/ Unit	(16) Insurance/ Unit
20 Units or Less	-133.8***	-146.9***	-141.1***	-138.1***	-21.54	-14.45	194.0***	200.6***
	(25.76)	(29.80)	(27.81)	(28.38)	(71.09)	(70.10)	(31.48)	(33.23)
21 - 60 Units	-35.58	-41.23	-95.12***	-93.80***	-78.23	-82.97	72.90***	74.00***
	(24.51)	(25.30)	(25.53)	(25.49)	(52.77)	(52.14)	(14.73)	(15.11)
Average Bedrooms Per Unit - Linear Term	-3.622	-57.65	26.87	71.63	55.74	307.6**	30.07**	97.24**
	(16.41)	(63.36)	(19.24)	(62.87)	(46.09)	(126.6)	(12.82)	(42.80)
Average Bedrooms Per Unit - Quadratic Term		17.32		-12.39		-64.90*		-18.58
		(15.69)		(15.00)		(37.53)		(12.27)
Number of Buildings - Linear Term	-0.164	-1.962	-0.135	-1.098	-3.744	-10.60**	-0.937	-1.382
	(0.876)	(1.918)	(1.111)	(2.522)	(2.537)	(5.070)	(0.766)	(1.632)
Number of Buildings - Quadratic Term		0.0460		0.0210		0.163*		0.00967
		(0.0312)		(0.0424)		(0.0902)		(0.0241)
LIHTC	-14.93	-11.81	12.67	12.25	-36.05	-28.82	-20.70	-20.37
	(15.98)	(15.83)	(23.60)	(23.70)	(45.08)	(44.60)	(15.29)	(15.30)
Percent PBS8 - Linear Term	62.71	-188.9*	104.4*	189.2	67.71	-577.6**	-42.27*	-97.20
	(42.11)	(108.4)	(54.47)	(151.0)	(93.96)	(290.7)	(24.97)	(87.98)
Percent PBS8 - Quadratic Term		276.5**		-92.83		716.7**		61.90
		(126.7)		(173.6)		(334.3)		(91.69)
Percent Affordable	28.00	22.07	-17.56	-14.56	-259.0***	-269.8***	29.89	29.30
	(33.12)	(32.32)	(37.29)	(37.36)	(74.11)	(74.90)	(31.71)	(31.77)
Interest Subsidy	-77.44*	-47.91	23.00	23.30	-5.748	13.48	-26.07	-22.84
	(45.39)	(32.47)	(25.22)	(25.10)	(63.25)	(62.92)	(22.71)	(22.81)
Section 8 Program	173.6**	-56.42	-5.148	3.455	271.9***	231.2**	30.44	28.88
	(79.98)	(53.67)	(57.16)	(61.97)	(103.1)	(107.3)	(27.33)	(27.66)
SHARP/RDAL	-144.0*	-106.1***	20.01	20.11	-59.75	-60.68	-10.81	-10.61
	(76.46)	(22.80)	(32.44)	(32.60)	(59.89)	(58.40)	(20.60)	(20.54)
SRO	-1.493	-93.48*	68.59	84.41	-493.7***	-414.0***	-112.0***	-91.69**
	(22.20)	(53.95)	(77.41)	(80.46)	(112.3)	(112.4)	(35.84)	(36.70)
Boston Market Location	-50.03	178.5**	51.44*	48.25*	123.4	145.5*	-48.60*	-44.99
	(32.27)	(79.69)	(27.09)	(28.25)	(84.86)	(85.64)	(27.06)	(27.66)
City of Boston Location	-34.17	-154.2**	145.4***	149.7***	155.6*	138.9	47.42*	47.03
	(49.06)	(77.06)	(35.72)	(35.54)	(87.17)	(86.29)	(28.37)	(28.75)
Gateway City Location	-105.1***	-5.953	46.77***	45.97***	-86.47**	-91.17**	-34.66**	-33.49**
	(22.62)	(23.75)	(15.25)	(15.99)	(39.69)	(40.98)	(14.69)	(14.98)
Constant	123.3***	178.8**	-0.363	-37.34	900.7***	716.3***	308.4***	253.3***
	(45.76)	(82.90)	(35.52)	(67.10)	(89.95)	(131.1)	(29.74)	(44.28)
Observations	625	625	625	625	625	625	625	625
R-squared	0.095	0.102	0.167	0.169	0.200	0.215	0.210	0.213

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

2008 Data

	(1) Admin./ Mgmt. Fee/Unit	(2) Admin./ Mgmt. Fee/Unit	(3) Maintenance/ Unit	(4) Maintenance/ Unit	(5) Utilities Net Water/Unit	(6) Utilities Net Water/Unit	(7) Water/Unit	(8) Water/Unit
20 Units or Less	-80.37	-106.7	123.5	44.17	-42.79	-95.77	-30.33	-52.58
	(125.3)	(132.3)	(156.9)	(163.9)	(110.0)	(116.7)	(49.21)	(52.42)
21 - 60 Units	61.49	50.16	188.8**	159.5*	-49.63	-71.97	-23.81	-29.22
	(67.74)	(68.30)	(88.58)	(88.93)	(65.37)	(65.81)	(29.36)	(29.43)
Average Bedrooms Per Unit - Linear Term	84.53	-316.4*	464.9***	-127.9	82.82	85.44	246.0***	197.5**
	(58.95)	(183.0)	(81.03)	(266.7)	(71.80)	(274.0)	(24.36)	(84.67)
Average Bedrooms Per Unit - Quadratic Term		111.0**		172.4**		12.00		16.71
		(48.14)		(74.15)		(86.49)		(25.42)
Number of Buildings - Linear Term	2.809	11.09	0.368	-0.653	-7.670*	-24.02***	-1.229	-6.835**
	(2.891)	(6.818)	(3.972)	(9.106)	(4.630)	(8.825)	(1.523)	(2.920)
Number of Buildings - Quadratic Term		-0.178		0.0481		0.392**		0.131***
		(0.123)		(0.140)		(0.198)		(0.0461)
LIHTC	178.4***	181.5***	37.96	49.36	186.0***	200.2***	-9.418	-7.498
	(63.26)	(63.51)	(73.23)	(72.27)	(61.92)	(61.03)	(27.93)	(28.37)
Percent PBS8 - Linear Term	512.6***	-241.4	351.7**	-788.8*	138.6	-650.0	-32.09	107.4
	(169.8)	(396.1)	(157.2)	(468.5)	(116.5)	(430.6)	(51.93)	(241.1)
Percent PBS8 - Quadratic Term		825.8*		1,248**		869.4*		-155.3
		(455.3)		(505.2)		(444.8)		(244.6)
Percent Affordable	-206.4	-232.9	-173.7	-206.6	-150.3	-160.6	-45.53	-39.77
	(141.7)	(141.5)	(152.6)	(152.8)	(119.7)	(120.6)	(55.42)	(56.12)
Interest Subsidy	3.397	0.946	222.9*	220.1*	859.0***	871.5***	121.7***	118.2***
	(101.5)	(99.71)	(114.4)	(113.2)	(96.57)	(97.12)	(44.74)	(45.32)
Section 8 Program	-30.16	-107.4	461.3**	344.4*	518.4***	454.4***	186.3***	195.1***
	(172.9)	(183.7)	(181.2)	(179.2)	(138.3)	(133.9)	(62.56)	(59.63)
SHARP/RDAL	-103.0	-102.6	316.2***	313.1***	420.5***	415.6***	200.3***	198.7***
	(85.27)	(86.10)	(115.4)	(117.5)	(93.69)	(92.75)	(45.28)	(45.46)
SRO	-696.3***	-837.7***	-578.0***	-767.4***	-313.0**	-296.0*	-55.19	-59.91
	(199.0)	(200.8)	(200.3)	(210.0)	(156.2)	(160.4)	(67.42)	(67.79)
Boston Market Location	360.4***	388.5***	548.7***	570.8***	348.5***	353.1***	232.3***	216.8***
	(124.3)	(122.7)	(209.7)	(214.7)	(106.1)	(104.9)	(67.94)	(66.18)
City of Boston Location	31.01	-8.641	-90.38	-147.4	157.0	131.2	39.50	44.49
	(126.0)	(125.3)	(213.8)	(216.7)	(107.8)	(104.9)	(69.48)	(67.76)
Gateway City Location	-84.02	-76.79	-92.36	-107.4	51.39	21.55	6.844	-5.704
	(64.73)	(65.24)	(76.58)	(80.01)	(57.95)	(60.78)	(27.16)	(27.60)
Constant	1,704***	2,036***	1,111***	1,640***	608.8***	666.5***	22.91	78.03
	(133.3)	(195.0)	(165.5)	(257.0)	(142.1)	(237.0)	(50.69)	(80.67)
Observations	612	612	612	612	612	612	612	612
R-squared	0.202	0.213	0.259	0.274	0.293	0.306	0.359	0.364

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

	(9) Services/Unit	(10) Services/Unit	(11) Security/Unit	(12) Security/Unit	(13) Taxes/Unit	(14) Taxes/Unit	(15) Insurance/ Unit	(16) Insurance/ Unit
20 Units or Less	-144.0***	-155.8***	-130.3***	-128.9***	79.29	88.48	216.8***	218.5***
	(25.38)	(29.64)	(27.41)	(27.21)	(74.20)	(74.43)	(34.14)	(36.38)
21 - 60 Units	-55.20**	-59.80***	-102.2***	-102.4***	-49.88	-54.47	73.50***	73.60***
	(22.01)	(23.07)	(20.83)	(20.78)	(54.91)	(54.31)	(17.82)	(18.22)
Average Bedrooms Per Unit - Linear Term	-2.042	-66.41	5.049	25.10	34.66	238.0*	20.05	19.86
	(16.28)	(65.37)	(15.94)	(49.75)	(45.49)	(127.1)	(13.53)	(40.86)
Average Bedrooms Per Unit - Quadratic Term		19.51		-5.309		-52.55		-0.155
		(16.27)		(11.67)		(36.63)		(11.57)
Number of Buildings - Linear Term	-0.389	-1.476	0.130	-0.104	-0.564	-4.411	-0.552	0.0787
	(0.903)	(2.131)	(0.976)	(2.172)	(2.511)	(5.172)	(0.880)	(1.896)
Number of Buildings - Quadratic Term		0.0287		0.00576		0.0942		-0.0143
		(0.0344)		(0.0373)		(0.0930)		(0.0294)
LIHTC	-5.783	-3.637	13.50	14.05	-17.02	-9.843	-30.80*	-30.72*
	(15.11)	(15.14)	(20.19)	(20.35)	(46.55)	(46.01)	(16.75)	(16.87)
Percent PBS8 - Linear Term	75.64*	-103.1	126.5**	60.80	67.37	-717.7***	-66.83**	-119.3
	(41.42)	(106.3)	(55.14)	(138.2)	(90.28)	(272.0)	(31.49)	(107.5)
Percent PBS8 - Quadratic Term		196.1		72.85		870.1***		57.93
		(122.7)		(153.0)		(308.2)		(115.2)
Percent Affordable	15.59	11.22	15.37	14.13	-238.6***	-253.3***	31.79	30.21
	(35.49)	(34.92)	(33.31)	(33.20)	(80.69)	(81.17)	(36.97)	(37.28)
Interest Subsidy	-68.72	-58.85*	21.89	23.56	42.00	60.72	-13.38	-12.62
	(41.89)	(30.21)	(26.26)	(25.84)	(66.63)	(66.25)	(27.54)	(27.74)
Section 8 Program	129.7*	-66.39	-63.82	-68.25	343.1***	288.6***	59.87*	55.69
	(69.79)	(51.80)	(55.02)	(58.51)	(102.3)	(105.3)	(34.43)	(35.52)
SHARP/RDAL	-112.1*	-114.7***	47.65	47.62	40.98	40.21	7.933	8.070
	(66.84)	(23.75)	(30.33)	(30.37)	(61.28)	(59.88)	(24.10)	(24.14)
SRO	-23.47	-88.40*	23.59	29.28	-480.9***	-422.2***	-176.8***	-178.4***
	(23.64)	(50.36)	(76.58)	(77.95)	(109.6)	(111.1)	(42.97)	(42.72)
Boston Market Location	-59.40**	132.6*	68.63**	71.41**	214.7**	245.1***	-2.269	0.681
	(30.05)	(69.97)	(29.25)	(30.08)	(89.81)	(90.78)	(29.63)	(29.94)
City of Boston Location	-49.24	-120.1*	94.05***	92.19***	94.93	72.00	12.75	10.69
	(47.96)	(67.55)	(35.12)	(34.57)	(94.11)	(93.53)	(30.63)	(30.82)
Gateway City Location	-114.0***	-26.88	47.82***	48.06***	-68.08*	-68.21	-23.66	-22.31
	(23.32)	(25.67)	(15.66)	(16.50)	(40.68)	(42.13)	(17.02)	(17.35)
Constant	148.0***	209.1**	17.62	2.257	773.0***	623.3***	317.2***	316.5***
	(46.03)	(85.39)	(28.68)	(52.89)	(87.53)	(134.5)	(32.66)	(45.90)
Observations	612	612	612	612	612	612	612	612
R-squared	0.088	0.093	0.175	0.176	0.204	0.218	0.191	0.191

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Quadratic Functions Interpretation

The following provides the quantitative interpretation for quadratic functions discussed in the narrative. The functions shown below meet the following criteria: i) the linear and quadratic terms are jointly significant; ii) the linear model and the quadratic model provide different interpretations, so that the linear model is not a good approximation of the quadratic model; and iii) the quadratic model provides greater predictive power than the linear model. For interpretation of other quadratic functions, the linear model can be used as an approximation. Appendix B provides additional information regarding quadratic functions.

Independent Variable: Average Number of Bedrooms Per Unit
Dependent Variable: Real Estate Taxes Per Unit

As compared to properties with zero bedrooms per unit, a property that has this number of average bedrooms per unit:		
1	2	3
is estimated to have this increased/decreased level of operating expenses per unit per 2009 data:		
$243	$356	$339
and this estimated increased/decreased level of operating expenses per unit per 2008 data:		
$186	$266	$242

Independent Variable: Percent Project-Based Section 8
Dependent Variable: Real Estate Taxes Per Unit

As compared to a baseline of a property with no Project-Based Section 8 units, this percentage of Project-Based Section 8 Units:									
10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
is estimated to have this increased/decreased level of operating expenses per unit per 2009 data:									
-$51	-$87	-$109	-$116	-$110	-$89	-$53	-$3	$61	$139
and this estimated increased/decreased level of operating expenses per unit per 2008 data:									
-$63	-$109	-$137	-$148	-$141	-$117	-$76	-$17	$59	$152

Appendix G: Detailed Summary of Independent Variable Findings

The following is a more detailed summary of independent variable findings including discussion of all operating expense categories where a statistically significant relationship was identified.[2] Coefficients representing the estimated magnitude of the relationship between independent and dependent variable respectively are included in order to provide an order of magnitude. These estimates should not be interpreted as precise measures of the relationship between a given independent variable and operating expense levels. Specific coefficients cited in the discussions related to overall operating expenses per unit are drawn from Appendix C. Appendix F provides the regression models for individual categories that are discussed. For the category regressions, quadratic regression model figures are used unless otherwise specified.[3]

All findings are expressed in 2009 dollars; 2008 data was adjusted to 2009 dollars using the Consumer Price Index for All Urban Consumers percentage change for December to December of 2.7%.

- **Number of Units**: Isolating a relationship between number of units and operating expenses proved difficult. Several forms of independent variables representing number of units (Number of Units as a continuous variable, Log Number of Units, and other dummy variables for unit range categories) were tested, and the categories 0-20 units, 21-60 units, and 60+ units provided the strongest predictive power. Still, these dummy variables are not jointly statistically significant. However, analysis of individual operating expense categories shines light on how unit count relates to operating expense levels:
    - **Administrative costs and management fees** per unit appear lower (estimated at $287 per units or $106 per unit) for properties of 21-60 units as compared to properties over 60 units. The difference is only statistically significant for the 2009 data.
    - **Utility** costs per unit appear lowest for properties under 20 units, higher for properties 21-60 units, and highest for properties over 60 units, but the relationships are not consistently statistically significant.
    - **Maintenance** costs per unit are estimated to be $148 or $160 per unit higher for properties 20-60 units as compared to larger properties.
    - **Services** costs for properties twenty units or less are an estimated $150 less costly per unit cost than other properties.
    - **Security** costs per unit escalate as project size increases. Properties 0-20 units have the lowest per unit security cost, while properties 60 units and higher have highest associated security costs.

[2] In the case of multiple dummy variables and variables where both linear and quadratic terms were employed, joint statistical significance was the threshold for inclusion. Unless otherwise specified, the 10% significance level is used to determine statistical significance.

[3] Linear functional form is used for ease of interpretation in cases where the quadratic form does not greatly improve predictive power.

  - **Insurance** cost per unit, as compared to properties over 60 units, is an estimated $200 or $218 per unit higher for properties under 20 units, and an estimated $ 74 for properties 21-60 units.

- **Average Bedrooms Per Unit**: The greater the number of average bedrooms per unit, the higher operating expenses per unit. The best way to model this variable utilizes a quadratic functional form. This form is employed because the data indicates that an increase from zero to one average bedrooms per units does not relate to the same delta on operating expenses per unit as increase from one to two average bedrooms per unit. Appendix C provides the precise associated increase in expenses per unit for each average bedroom size that is indicated by the data. Average Bedrooms Per Unit is related to several operating expense categories:
  - **Administrative costs and management fees** increase as average bedrooms per unit increases. The linear model is a good approximate fit here; it indicates that the increase per bedroom is estimated at $108 or $85 per unit. The association is statistically significant only in the 2009 data set.
  - **Maintenance** cost per unit increase with each additional average bedroom. The linear models estimate the increase to be $487 or $565.
  - **Water** expense is highly correlated with average bedroom density. The linear regressions models estimates for the association are $246 and $253 per unit.
  - **Real estate taxes** have a quadratic relationship to average bedroom size. See Appendix F for detail.
  - **Insurance** per unit is associated with an estimated $20 or $30 additional per unit for each additional average bedroom per the linear regression models.

- **Number of Buildings:** The data indicates that each additional building is associated with a lower level of per unit operating expenses. A quadratic functional form is used because the magnitude of the change in operating expenses associated with one additional building decreases slightly the higher the number of buildings. A roughly $40 less in operating expenses per unit is seen for each additional building. This finding is similarly present in the MassHousing data; the MHP data, however, shows a much smaller magnitude for the relationship between number of buildings and operating expenses that is not statistically significant. While there are some statistically significant relationships with individual operating expense categories, the magnitudes are very small (many less than $5 per unit; all less than $15 per unit) and lack practical significance; therefore, they will not be discussed.

- **LIHTC Status:** LIHTC status may be associated with increased operating expense levels, though there is a lack of consistency across the portfolio data sets. A larger and statistically significant elevation in expenses association with LIHTC status is seen in the MHP data while the MassHousing data shows a small, non-statistically significant correlation. These disparate results indicate that distinct

characteristics common to each portfolio that are not included as control variables create biases. While the magnitude of the increase associated with LIHTC status is difficult to quantify from available data, analysis of categories reveals some insight as to why LIHTC status appears correlated with elevated per unit operating expense levels.

- **Administrative costs and management fees** are higher for LIHTC projects (estimates $155 and $182 per unit.)
- **Utility** costs also appear higher for LIHTC projects (estimated at $115 and $200 per unit.)

- **Percent Project-Based Section 8:** Greater proportions of Project-Based Section 8 units are associated with elevated operating expense levels. The quadratic model provides the best fit to describe this relationship. As the proportion of Project-Based Section 8 units increases, the impact of an incremental increase in Section 8 units becomes larger in magnitude. The relationship of Project-Based Section 8 units and operating expenses can be seen across several operating expense categories:
  - **Administrative costs and management fees** increase as the proportion of Project-Based Section 8 units increases. Costs increase an estimated $4 to $5 per unit with each percentage point increase in Project-Based Section 8 units, using the linear models.
  - **Maintenance** expense increases with additional proportions of Project-Based Section 8 units. The linear models indicates that a project without Project-Based Section 8 would be an estimated $352 or $368 less per unit to operate than a 100% Project-Based Section 8 property.
  - **Services** cost may have a positive association with proportion of Project-Based Section 8 units. However, the increase is modest; the linear models (while not consistently statistically significant) indicate that 100% Project-Based Section 8 properties are about $48 or $75 more costly in services than projects with no Project-Based Section 8 units.
  - **Real estate taxes** have a quadratic functional relationship with the proportion of Project-Based Section 8 units. The regressions indicate that the least expensive per unit real estate taxes are paid for properties of around 50% Project-Based Section 8, while the most expensive properties are 90-100% Section 8. See Appendix F for detail.

- **Percent Affordable:** There is no statistically significant relationship between the proportion of units affordable and operating expenses apparent in the combined data or the MHP data. It is possible, however, that there is omitted variable bias that distorts the findings as the MassHousing data does show a statistically significant $14 to $15 dollar per unit decrease in per unit expense levels for each percentage point increase in affordable units. There is one operating expense category where a statistically significant relationship to percent affordable was found:
  - **Real estate taxes** are estimated to increase by $2 to $3 per unit for each percentage point increase in units affordable.

- **SRO Status:** SROs are cheaper to operate than other properties on a per unit basis. (Note that this holds true when average bedroom size is included as a control.) The combined data set shows estimates of $2,118 and $2,153 for the decrease in per unit expenses for SROs, though the MHP data shows smaller magnitudes. It should be noted that there are no SROs in the MassHousing portfolio so the data on SROs comes entirely from MHP.
    - **Administrative costs and management fees** are estimated at $639 or $837 less per unit for SRO projects.
    - **Maintenance** costs per units are similarly lower; estimates of this magnitude are $767 and $823.
    - **Services** costs are slightly less per unit for SRO projects; estimates of the delta are $88 or $93 per unit.
    - **Real estate taxes** are an estimated $414 or $422 per unit lower for SRO projects.
    - **Insurance** per unit costs less for SROs, though the magnitude of estimates ranges from $92 to $179 per unit.

- **Boston Market Location:** Location in the Boston Market is associated with an elevated level of operating expenses per unit. The combined data indicates the premium to be $1,407 to $1,578. This finding is consistently statistically significant across portfolios, though magnitude estimates range to $1,790.
    - **Administrative costs and management fees** are higher for projects in the Boston Market; estimates from the linear models are $364 and $388 per unit.
    - **Maintenance** costs are estimated by the linear models to be $571 or $401 higher per unit for Boston Market projects.
    - **Utility** costs are estimated by the linear models to be $353 or $334 higher per unit for projects located in the Boston Market.
    - **Water** expense per unit is estimated at $217 and $214 per unit more for Metro Boston projects.
    - **Services** costs are higher for Boston Market projects; estimates of the magnitude are $179 and $133.
    - **Security** cost is estimated to be $48 or $71 per unit higher per unit for projects located in the Boston Market.
    - **Real estate taxes** per unit are estimated to be $146 or $245 higher for Boston Market projects.

- **City of Boston Location:** The combined data set indicates that location in the City of Boston itself is associated with additional elevated costs above the Boston Market levels. However, the data sets do not show a consistent nor statistically significant distinction. A few expense categories, however, reveal a distinction between the City of Boston and the rest of the Boston Market:
    - **Utility** costs may be higher for projects in the City of Boston. The 2009 data indicates a statistically significantly elevated level of utility costs of $188 per unit over other Boston Market projects, or a $522 more than

projects not in the Boston Market. The 2009 data shows an elevated level of utility costs for Boston projects, but the association is not statistically significant.
    - **Services** costs are lower than for other projects in the Boston Market. Boston projects spend only an estimated $13 or $25 more per unit in services than projects outside the Boston Market, which is $154 or $120 per unit less than projects in Brookline, Cambridge, and Somerville.
    - **Security** costs are higher in Boston proper; the magnitude is estimated at $92 or $150.

- **Gateway City Location:** Location in a Gateway City appears to be associated with a decreased level of operating expenses per unit, though the relationship is less strong than that of the Boston Market variable. The combined data set regressions estimate the association to be $260 or $285 per unit. There is a negative relationship seen across the individual agency portfolios, with a range of $257 (MassHousing portfolio) to $315 (MHP portfolio) per unit in magnitude. The expense categories where Gateway City location appear significant are:
    - **Administrative costs and management fees** are an estimated $74 or $82 per unit less than for other projects.
    - **Maintenance** costs may be lower for Gateway City projects. The 2009 data shows a statistically significant association of $185 per unit less in maintenance expense for Gateway City projects. The 2008 data shows a smaller delta in maintenance costs for Gateway City projects that is not statistically significant.
    - **Security** is estimated at $46 or $48 per unit more for Gateway City properties.
    - **Real estate taxes** may be lower for Gateway City projects. The 2009 data shows a statistically significant negative relationship of $91 per unit, while the 2008 data shows a non-statistically significant association of $68 per unit.
    - **Insurance** may be lower for Gateway City projects. The 2009 data shows a statistically significant negative relationship of $33 per unit, while the 2008 data shows a non-statistically significant association of $22 per unit.

- **Primary Program Type:** No consistent, statistically significant relationship between overall expenses per unit and primary program type is seen in the regression models. The inclusion of program type does, however, notably alter the magnitude of other independent variable coefficients, indicating that controlling for program type is useful for accurately isolating the association between various independent variables and expenses. A few expense categories show a statistically significant relationship with one or more program types:
    - **Administrative costs and management fees** appear lower for SHARP/RDAL projects by an estimated $103 or $265 per unit. This finding is statistically significant only for the 2009 data.
    - **Maintenance** expense findings are inconsistent for Interest Subsidy and Project-Based Section 8 program types, but data shows a consistently

elevated level of per unit expense for SHARP/RDAL as compared to other projects.

- **Service** costs are lower for SHARP/RDAL, estimated at $115 or $106 per unit.
- **Utilities** are higher for Interest Subsidy, Section 8 Program, and SHARP/RDAL projects as compared to other projects. Estimates of the magnitude of this increase per unit are: 1) Interest Subsidy: $892 and $593; 2) Section 8 Program: $454 and $225; and 3) SHARP/RDAL: $416 and $341.
- **Water** costs are higher for Interest Subsidy, Section 8 Program, and SHARP/RDAL projects as compared to other projects. Estimates of the magnitude of this increase per unit are: 1) Interest Subsidy: $118 and $1073; 2) Section 8 Program: $195 and $133; and 3) SHARP/RDAL: $129 and $199.

- **Presence of Services:** The combined data set indicates that presence of services is associated with a $26 or $341 increase in operating expenses per unit. Use of these variables improves predictive power of the regressions, but the magnitude varies across data sets ($688 to $958 for MHP and $206 to $285 for MassHousing.)

- **Presence of Security:** Again, inclusion of this variable improves the predictive power of the regression, but magnitude is unclear and coefficients are not statistically significant for the combined data or the MHP data. The combined data indicates an association of about $35 or $45 added expense per unit, while MHP's data shows a large range of $46 to $367. Interestingly, there is a highly statistically significant relationship in the MassHousing data of a much larger magnitude: $756 or $763 per unit.

Appendix H
2009 MHP and MassHousing Portfolio Summary Statistics

	MHP		MassHousing	
	Statistic	**Standard Deviation**	**Statistic**	**Standard Deviation**
PORTFOLIO STATISTICS				
Average Number of Units	46.10	51.56	124.05	98.96
Median Number of Units	33		103	
% 20 Units or Less	36.31%		4.83%	
% 21-60 Units	42.26%		19.55%	
% 60+ Units	21.43%		75.62%	
Average Bedrooms Per Unit	1.80	0.76	1.58	0.59
Average Number of Buildings	4.45	5.17	6.58	8.94
% LIHTC	53.57%		36.69%	
Average % Affordable	65.09%	0.37	83.24%	0.28
Average % Project-Based Section 8	18.51%	0.34	44.54%	0.47
% Interest Subsidy Program	3.00%		28.78%	
% Section 8 Program	0.00%		41.42%	
% SHARP/RDAL	0.00%		14.28%	
% SRO	11.90%		0.00%	
% Located in City of Boston	34.52%		29.64%	
% Located in Boston Market	41.07%		33.27%	
% Located in Gateway City	31.55%		31.45%	
% with Services	26.79%		63.31%	
% with Security	35.12%		61.29%	
SUMMARY OF DEPENDENT VARIABLES				
Average Operating Expenses/Unit	$ 7,258	$ 2,334	$ 7,842	$ 1,928
Median Operating Expenses/Unit	$ 7,072		$ 7,589	
Average Admin. + Mngmt Fee /Unit	$ 2,026	$ 893	$ 2,128	$ 629
Average Maintenance/Unit	$ 2,299	$ 1,055	$ 2,336	$ 924
Average Utilities/Unit	$ 819	$ 659	$ 1,253	$ 554
Average Water/Unit	$ 520	$ 369	$ 574	$ 310
Average Services/Unit	$ 93	$ 209	$ 124	$ 284
Average Security/Unit	$ 63	$ 229	$ 135	$ 266
Average Taxes/Unit	$ 754	$ 445	$ 933	$ 547
Average Insurance/Unit	$ 436	$ 250	$ 361	$ 144
	Sample size = 168		Sample size = 496	

Appendix I: MHP Data Set Regressions

This regression table shows several versions of regressions performed to analyze the relationship between the independent variables and operating expenses per unit for the MHP data set. Linear regressions both including and excluding the variables Services and Security (variables hoped to be useful for underwriting purposes, but acknowledged to be categories of the dependent variable) are provided as reference.

Regressions (4) and (10) demonstrate that use of the variable New or Rehab in Last 10 Years is statistically significant. This variable provides a slightly less powerful predictive fit than the combination of Age (Linear and Quadratic) variables, however, and therefore the Age variable forms are used in the best fit model.

In contrast to the findings for the combined data set, for the MHP data set, Percent Affordable – Quadratic Term was jointly significant with Percent Affordable – Linear Term and improved the predictive power of the regression.

Neither the variable Property Condition representing the property quality grade nor the dummy variable Property B or Better is individually significant with or without the inclusion of the other variable. Both are included in Regressions (5) and (11) because, though not statistically significant, each improves the predictive power of the overall regression.

As was the case for the combined data set, the log-level regression form where Log Operating Expenses Per Unit is used as the dependent variable provides a slightly less robust predictive form, though only marginally so for the 2009 data. Further, some variables, namely Age (Linear and Quadratic) are found to be better fit by the log-level form.

Regression descriptions:

- (1) and (7): Linear regression of variables in combined data set only; excludes Services and Security
- (2) and (8): Linear regression of variables in combined data set only; includes Services and Security
- (3) and (9): Quadratic terms added to match Appendix B (Best predictive fit model for combined data set)
- (4) and (10): Same as above, but with addition of New or Rehab in Last 10 Years
- (5) and (11): Includes all MHP variables for best predictive model
- (6) and (12): Same as above but with Log Operating Expenses Per Unit as dependent variable

2009 Data

	(1) Op. Ex/Unit	(2) Op. Ex/Unit	(3) Op. Ex/Unit	(4) Op. Ex/Unit	(5) Op. Ex/Unit	(6) Log Op. Ex./Unit
20 Units or Less	-341.3	-7.092	-69.88	-44.42	-281.3	-0.0690
	(412.1)	(457.7)	(476.1)	(471.6)	(508.3)	(0.0761)
21 - 60 Units	187.1	227.3	235.4	222.6	13.69	0.000270
	(303.2)	(325.3)	(334.5)	(329.2)	(365.3)	(0.0518)
Average Bedrooms Per Unit - Linear Term	904.9***	953.1***	-280.9	-283.7	-471.8	-0.000342
	(274.0)	(274.5)	(1,101)	(1,068)	(1,176)	(0.173)
Average Bedrooms Per Unit - Quadratic Term			342.7	337.8	307.3	0.0218
			(323.4)	(317.0)	(365.1)	(0.0518)
Number of Buildings - Linear Term	-15.26	-23.89	-24.34	-37.49	-12.44	-0.000169
	(23.98)	(21.73)	(54.53)	(51.73)	(58.28)	(0.00843)
Number of Buildings - Quadratic Term			-0.0758	0.382	-0.369	-6.41e-05
			(1.522)	(1.477)	(1.637)	(0.000235)
Average Square Feet Per Unit					1.029	0.000212
					(1.039)	(0.000136)
Property Condition					115.9	0.00898
					(203.2)	(0.0307)
Property B or Better					11.15	-0.0134
					(433.7)	(0.0689)
Age - Linear Term					-300.1	-0.0609**
					(199.9)	(0.0265)
Age - Quadratic Term					18.76*	0.00352**
					(11.30)	(0.00148)
New or Rehab in Last 10 Years				-787.2**		
				(325.9)		
LIHTC	1,117***	1,133***	1,139***	1,725***	1,088***	0.163***
	(298.6)	(299.0)	(304.5)	(401.5)	(330.0)	(0.0448)
Percent PBS8 - Linear Term	1,218***	1,126**	1,502	1,896	1,269	0.164
	(451.7)	(434.7)	(1,578)	(1,595)	(1,570)	(0.207)
Percent PBS8 - Quadratic Term			-480.3	-979.0	-363.6	-0.0412
			(1,691)	(1,701)	(1,677)	(0.225)
Percent Affordable - Linear Term	402.4	438.2	499.9	722.7*	2,426**	0.332**
	(378.8)	(370.8)	(374.7)	(377.4)	(1,076)	(0.165)
Percent Affordable - Quadratic Term					-1,298	-0.198
					(785.9)	(0.121)
Interest Subsidy Program	36.74	-34.48	-712.4	-1,060	-629.9	0.0137
	(850.0)	(879.0)	(915.5)	(839.3)	(1,062)	(0.152)
Non-Profit Developer					216.1	0.0657
					(292.4)	(0.0440)
Management Quality					50.29	0.0155
					(139.1)	(0.0223)
Self-Managed					-392.5	-0.0674
					(282.5)	(0.0419)
Vacancy					47.49	0.00865
					(38.30)	(0.00566)
SRO	-1,566***	-1,489***	-1,893***	-1,910***	-1,873***	-0.249**
	(501.8)	(501.6)	(553.9)	(538.7)	(687.8)	(0.108)
Boston Market Location	1,979***	1,579***	1,535***	1,464***	1,211***	0.134**
	(337.5)	(344.8)	(342.9)	(341.3)	(352.0)	(0.0576)
City of Boston Location	-280.7	-315.7	-314.7	-340.8	-236.9	-0.0409
	(277.7)	(272.5)	(269.8)	(264.0)	(271.8)	(0.0406)
Gateway City Location	-222.5	-237.2	-296.2	-365.7	-406.9	-0.0643
	(323.3)	(300.1)	(314.1)	(311.8)	(309.2)	(0.0475)
Security		364.8	367.2	458.1	332.6	0.0355
		(310.6)	(313.7)	(322.1)	(329.5)	(0.0474)
Services		936.7***	958.1***	987.7***	899.3**	0.104**
		(310.4)	(328.9)	(331.0)	(351.9)	(0.0470)
Constant	4,198***	3,791***	4,783***	4,709***	4,432*	8.450***
	(531.2)	(566.1)	(954.7)	(930.9)	(2,272)	(0.323)
Observations	168	168	168	168	168	168
R-squared	0.567	0.599	0.603	0.614	0.644	0.643

Robust standard errors in p
*** p<0.01, ** p<0.05, * p<0.1

2008 Data

	(7) Op. Ex/Unit	(8) Op. Ex/Unit	(9) Op. Ex/Unit	(10) Op. Ex/Unit	(11) Op. Ex/Unit	(12) Log Op. Ex./Unit
20 Units or Less	648.7	782.2	707.5	765.1	645.8	0.170
	(550.8)	(581.1)	(599.6)	(594.7)	(592.2)	(0.142)
21 - 60 Units	376.6	346.1	354.8	358.5	190.0	0.0854
	(389.1)	(411.4)	(413.3)	(406.4)	(372.4)	(0.0838)
Average Bedrooms Per Unit - Linear Term	937.2***	945.5***	33.48	19.83	803.1	0.229
	(287.2)	(286.3)	(1,220)	(1,186)	(1,156)	(0.191)
Average Bedrooms Per Unit - Quadratic Term			256.9	252.9	56.48	-0.0218
			(351.3)	(346.4)	(359.7)	(0.0549)
Number of Buildings - Linear Term	5.846	-1.767	-3.761	-14.04	-7.493	0.00442
	(23.60)	(22.64)	(59.22)	(55.61)	(59.72)	(0.0106)
Number of Buildings - Quadratic Term			0.0207	0.418	-0.309	-0.000161
			(1.555)	(1.487)	(1.517)	(0.000276)
Average Square Feet Per Unit					-0.153	6.49e-06
					(1.162)	(0.000162)
Property Condition					59.19	-0.0276
					(220.0)	(0.0444)
Property B or Better					-94.48	-0.0650
					(467.7)	(0.0910)
Age - Linear Term					297.7	0.0758
					(265.5)	(0.0746)
Age - Quadratic Term					-9.918	-0.00284
					(14.05)	(0.00348)
New or Rehab in Last 10 Years				-779.1**		
				(355.3)		
LIHTC	1,871***	1,882***	1,872***	2,451***	2,131***	0.389***
	(382.7)	(397.7)	(407.7)	(481.0)	(422.5)	(0.122)
Percent PBS8 - Linear Term	1,462***	1,411***	755.9	1,205	723.8	0.183
	(435.4)	(423.2)	(1,795)	(1,829)	(1,712)	(0.264)
Percent PBS8 - Quadratic Term			658.9	107.7	574.9	-0.00670
			(1,915)	(1,941)	(1,852)	(0.287)
Percent Affordable - Linear Term	246.8	341.1	377.4	593.5	4,193***	0.676***
	(392.0)	(390.8)	(389.4)	(386.0)	(1,216)	(0.245)
Percent Affordable - Quadratic Term					-2,618***	-0.448**
					(873.0)	(0.175)
Interest Subsidy Program	523.6	458.2	64.61	-286.6	695.6	0.228
	(949.5)	(980.9)	(1,113)	(1,041)	(1,069)	(0.183)
Non-Profit Developer					415.8	0.120*
					(309.8)	(0.0630)
Management Quality					42.17	0.0208
					(144.6)	(0.0235)
Self-Managed					-629.7*	-0.142*
					(320.8)	(0.0833)
Vacancy					46.29	0.00329
					(41.10)	(0.00838)
SRO	-1,236**	-1,255**	-1,566**	-1,572**	-1,025	0.00855
	(613.7)	(628.5)	(712.1)	(694.7)	(863.8)	(0.204)
Boston Market Location	2,048***	1,816***	1,790***	1,720***	1,143***	0.0654
	(359.1)	(367.5)	(365.7)	(365.3)	(395.5)	(0.0888)
City of Boston Location	-243.0	-251.2	-245.7	-284.8	32.61	-0.0424
	(317.9)	(321.9)	(323.2)	(319.7)	(286.4)	(0.0528)
Gateway City Location	-306.3	-258.1	-314.9	-371.9	-617.0*	-0.0980*
	(306.9)	(299.9)	(312.8)	(306.3)	(323.8)	(0.0541)
Security		42.87	46.38	149.1	143.9	0.0198
		(294.3)	(294.3)	(308.6)	(327.7)	(0.0503)
Services		720.8**	688.0*	710.6*	515.8	0.00470
		(350.5)	(365.3)	(369.0)	(395.4)	(0.0904)
Constant	3,153***	2,969***	3,748***	3,658***	354.6	7.602***
	(752.1)	(770.4)	(1,189)	(1,164)	(2,428)	(0.459)
Observations	157	157	157	157	157	157
R-squared	0.594	0.606	0.609	0.619	0.669	0.580

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Appendix J: MassHousing Data Set Regressions

This regression table shows several versions of regressions performed to analyze the relationship between the independent variables and operating expenses per unit for the MassHousing data set. Linear regressions both including and excluding the variables Services and Security (variables hoped to be useful for underwriting purposes, but acknowledged to be categories of the dependent variable) are provided as reference.

Interestingly, use of Log Number of Units provided a better fit for the MassHousing data than did unit range dummy variables as demonstrated by the difference in R^2 values between Regressions (3) and (4) and (11) and (12) respectively.

In contrast to the findings for the combined data set, for the MassHousing data set, Percent Affordable – Quadratic Term was jointly significant with Percent Affordable – Linear Term and improved the predictive power of the regression. Coefficients can become distorted when one or more independent variables are highly correlated with other independent variables; this is the case with Percent Low Income, Percent Moderate Income and Percent Affordable. When all of these variables are included together, as in Regressions (7) and (15), the best overall predictive fit is achieved. Individual coefficients on the variables, however, can be distorted. Therefore, Regressions (6) and (14) omit Percent Low Income (Linear and Quadratic) and Percent Moderate Income (Linear and Quadratic) to provide a better approximation of the coefficients on the other variables.

As was the case for the combined data set, the log-level regression form where Log Operating Expenses Per Unit is used as the dependent variable provides a slightly less robust predictive form.

Regression descriptions:

- (1) and (9): Linear regression of variables in the combined data set only; excludes Services and Security
- (2) and (10): Linear regression of variables in the combined data set only; includes Services and Security
- (3) and (11): Quadratic terms added to match Appendix B (Best predictive fit model for combined data set)
- (4) and (12): Same as above but Log Number of Units replaces unit range dummy variables
- (5) and (13): Same as above but Percent Low Income - Linear and Percent Low Income – Quadratic replace Percent Affordable
- (6) and (14): Includes all variables except Percent Low Income (Linear and Quadratic) and Percent Moderate Income (Linear and Quadratic)
- (7) and (15): Includes all MassHousing variables for best predictive model
- (8) and (16): Same as above but with Log Operating Expenses Per Unit as dependent variable

2009 Data

	(1) Op. Ex./Unit	(2) Op. Ex./Unit	(3) Op. Ex./Unit	(4) Op. Ex./Unit	(5) Op. Ex./Unit	(6) Op. Ex./Unit	(7) Op. Ex./Unit	(8) Log Op. Ex//Unit
20 Units or Less	472.0	950.5***	826.8**					
	(341.5)	(348.1)	(341.4)					
21 - 60 Units	122.2	305.3*	245.8					
	(184.1)	(177.8)	(178.5)					
Log Number of Units				-320.5***	-291.7***	-508.7***	-512.9***	-0.0663***
				(110.0)	(106.9)	(128.9)	(128.5)	(0.0161)
Average Bedroms Per Unit - Linear Term	1,059***	1,025***	1,101*	1,085*	1,044*	919.0	847.2	0.164*
	(163.4)	(155.1)	(611.0)	(629.4)	(624.6)	(713.0)	(712.0)	(0.0918)
Average Bedroms Per Unit - Quadratic Term			10.30	6.304	18.54	60.13	78.51	-0.00818
			(184.6)	(189.8)	(186.5)	(195.1)	(195.1)	(0.0236)
Number of Buildings - Linear Term	-8.888	-15.63*	-40.21**	-32.67*	-32.31*	-5.723	-5.178	-0.00170
	(9.496)	(9.373)	(18.21)	(18.80)	(19.18)	(20.76)	(20.85)	(0.00271)
Number of Buildings - Quadratic Term			0.566*	0.540*	0.489	0.185	0.172	3.33e-05
			(0.291)	(0.312)	(0.326)	(0.267)	(0.270)	(3.62e-05)
LIHTC	-34.16	-44.25	-7.269	8.281	35.39			
	(146.0)	(140.0)	(140.4)	(140.5)	(145.5)			
9% LIHTC						61.62	72.59	0.0146
						(150.1)	(150.7)	(0.0199)
4% LIHTC						-278.8	-209.2	-0.0266
						(182.5)	(194.4)	(0.0245)
Percent PBS8 - Linear Term	1,171**	1,225***	-1,205	-1,106	-222.3	-448.9	-43.90	0.0303
	(482.7)	(459.2)	(1,036)	(1,046)	(1,113)	(1,156)	(1,219)	(0.159)
Percent PBS8 - Quadratic Term			2,699**	2,633**	2,229*	1,804	1,646	0.156
			(1,105)	(1,118)	(1,241)	(1,168)	(1,319)	(0.171)
Percent Affordable - Linear Term	-1,190***	-1,411***	-1,561***	-1,660***		-2,780	-18,753	-4.864
	(358.6)	(365.1)	(371.8)	(371.2)		(1,960)	(38,177)	(6.044)
Percent Affordable - Quadratic Term						1,172	996.7	0.0630
						(1,528)	(1,593)	(0.208)
Percent Low Income - Linear Term					-1,742		14,257	4.329
					(1,323)		(38,233)	(6.046)
Percent Low Income - Quadratic Term					315.9		1,431	0.221
					(1,180)		(1,709)	(0.219)
Percent Moderate Income - Linear Term							18,275	4.922
							(38,216)	(6.044)
Percent Moderate Income - Quadratic Term							-2,197	-0.317
							(1,472)	(0.194)
Interest Subsidy Program	244.7	335.1	433.0*	458.5*	-374.4*	71.69	-257.0	-0.0501
	(234.3)	(244.7)	(248.5)	(250.6)	(193.1)	(266.8)	(325.6)	(0.0433)
Section 8 Program	470.9	448.7	313.3	295.6	-234.8	344.6	166.8	0.0298
	(475.5)	(458.7)	(458.1)	(460.3)	(465.2)	(479.3)	(483.6)	(0.0582)
SHARP/RDAL	-213.8	-245.9	-255.0	-297.1	-306.6	-327.8	-355.7	-0.0550
	(227.9)	(220.7)	(220.0)	(217.1)	(228.8)	(268.7)	(272.4)	(0.0367)
Percent Elderly						-409.8	-394.9	-0.0616*
						(250.0)	(252.8)	(0.0341)
Boston Market Location	1,460***	1,451***	1,526***	1,553***	1,546***	1,457***	1,414***	0.193***
	(420.2)	(436.2)	(423.1)	(419.2)	(419.5)	(463.9)	(462.3)	(0.0529)
City of Boston Location	632.3	502.7	426.8	401.6	368.3	175.2	234.5	0.00370
	(428.8)	(439.5)	(418.9)	(413.9)	(417.2)	(445.4)	(441.9)	(0.0500)
Gateway City Location	-50.14	-223.9	-262.5*	-229.7	-296.3*	-376.2**	-368.5**	-0.0478**
	(157.2)	(152.9)	(158.9)	(159.1)	(158.0)	(167.9)	(168.7)	(0.0219)
Average Stories Per Building						44.43	45.47	0.00536
						(28.08)	(28.55)	(0.00344)
Number of Elevators						52.01	44.54	0.00697
						(60.10)	(60.67)	(0.00775)
Scattered Site						-315.6**	-309.2*	-0.0417**
						(160.2)	(159.3)	(0.0208)
Concrete						95.50	97.15	0.00400
						(210.5)	(211.7)	(0.0268)
Steel						171.3	169.9	0.0186
						(183.1)	(185.0)	(0.0237)
Heat Individually Metered						-410.1*	-393.9*	-0.0523*
						(224.9)	(225.1)	(0.0301)
Electricity Individually Metered						-424.2***	-417.8***	-0.0529***
						(155.5)	(156.6)	(0.0197)
Gas Heat						-88.20	-91.86	-0.0140
						(181.8)	(184.5)	(0.0244)
Electric Heat						-92.20	-185.4	-0.0300
						(285.5)	(290.2)	(0.0394)
Services		291.0**	284.9**	309.4**	277.2**	324.9**	317.4**	0.0396**
		(139.0)	(136.4)	(132.1)	(134.5)	(136.0)	(136.8)	(0.0182)
Security		747.0***	763.4***	809.4***	803.6***	791.2***	797.1***	0.110***
		(136.8)	(136.4)	(135.9)	(136.9)	(134.4)	(135.1)	(0.0181)
Constant	5,721***	5,366***	5,415***	6,962***	6,743***	8,430***	8,876***	9.050***
	(323.9)	(318.6)	(542.9)	(734.9)	(753.2)	(1,000)	(1,024)	(0.136)
Observations	451	451	451	451	451	434	434	434
R-squared	0.463	0.499	0.508	0.512	0.507	0.542	0.546	0.534

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

2008 Data

	(9) Op. Ex./Unit	(10) Op. Ex./Unit	(11) Op. Ex./Unit	(12) Op. Ex./Unit	(13) Op. Ex./Unit	(14) Op. Ex./Unit	(15) Op. Ex./Unit	(16) Log Op. Ex//Unit
20 Units or Less	183.1	625.3	504.4					
	(405.7)	(400.8)	(416.3)					
21 - 60 Units	291.1	465.5**	401.2**					
	(198.7)	(192.3)	(193.0)					
Log Number of Units				-332.7***	-312.4***	-469.7***	-470.4***	-0.0565***
				(116.0)	(111.5)	(144.1)	(143.5)	(0.0171)
Average Bedroms Per Unit - Linear Term	1,073***	1,037***	1,260**	1,300**	1,259**	1,187*	1,130	0.184**
	(165.0)	(157.7)	(624.9)	(622.5)	(619.2)	(696.9)	(697.2)	(0.0878)
Average Bedroms Per Unit - Quadratic Term			-28.22	-55.98	-44.35	-16.69	0.963	-0.0137
			(184.3)	(183.3)	(180.6)	(193.3)	(194.8)	(0.0234)
Number of Buildings - Linear Term	-10.07	-15.97*	-45.12**	-35.58*	-34.13*	-6.120	-4.640	-0.00140
	(9.425)	(9.306)	(18.16)	(18.77)	(19.17)	(21.32)	(21.62)	(0.00274)
Number of Buildings - Quadratic Term			0.668**	0.613**	0.551*	0.203	0.175	2.95e-05
			(0.277)	(0.288)	(0.302)	(0.272)	(0.275)	(3.51e-05)
LIHTC	-20.84	-21.51	21.27	43.53	71.79			
	(146.9)	(142.4)	(141.7)	(142.1)	(148.3)			
9% LIHTC						126.6	135.1	0.0204
						(161.3)	(161.2)	(0.0202)
4% LIHTC						-270.9	-224.3	-0.0256
						(183.9)	(198.5)	(0.0231)
Percent PBS8 - Linear Term	1,156***	1,224***	-1,585*	-1,530*	-642.7	-978.4	-381.7	0.0162
	(438.3)	(428.1)	(912.8)	(923.5)	(1,033)	(1,078)	(1,161)	(0.147)
Percent PBS8 - Quadratic Term			3,127***	3,102***	2,627**	2,449**	2,021	0.171
			(948.7)	(958.7)	(1,147)	(1,078)	(1,276)	(0.159)
Percent Affordable - Linear Term	-1,012**	-1,256***	-1,427***	-1,472***		-2,340	-10,573	-3.640
	(401.6)	(408.9)	(417.1)	(426.6)		(2,109)	(48,456)	(6.463)
Percent Affordable - Quadratic Term						937.7	503.1	0.00440
						(1,653)	(1,701)	(0.214)
Percent Low Income - Linear Term					-1,928		6,200	3.116
					(1,365)		(48,376)	(6.452)
Percent Low Income - Quadratic Term					595.8		2,019	0.303
					(1,247)		(1,879)	(0.233)
Percent Moderate Income - Linear Term							10,550	3.758
							(48,075)	(6.422)
Percent Moderate Income - Quadratic Term							-1,853	-0.268
							(1,634)	(0.209)
Interest Subsidy Program	236.7	341.4	461.2	459.2	-294.6	-0.930	-271.7	-0.0561
	(284.3)	(298.5)	(306.2)	(312.1)	(225.1)	(323.1)	(364.5)	(0.0455)
Section 8 Program	443.5	436.0	282.9	254.0	-225.3	160.2	13.15	-0.000176
	(455.5)	(453.9)	(441.8)	(445.5)	(445.6)	(465.8)	(471.2)	(0.0557)
SHARP/RDAL	-247.0	-297.4	-299.6	-319.2	-334.8	-420.7	-418.0	-0.0631
	(260.8)	(257.7)	(256.9)	(254.1)	(263.8)	(306.4)	(306.9)	(0.0394)
Percent Elderly						-321.2	-289.5	-0.0394
						(249.8)	(252.6)	(0.0322)
Boston Market Location	1,685***	1,689***	1,784***	1,829***	1,823***	1,793***	1,763***	0.230***
	(477.2)	(488.6)	(473.5)	(466.3)	(465.5)	(527.2)	(530.0)	(0.0556)
City of Boston Location	460.9	322.4	235.2	181.7	146.3	-91.01	-44.59	-0.0314
	(490.7)	(496.0)	(473.8)	(464.3)	(465.4)	(509.6)	(510.7)	(0.0534)
Gateway City Location	-50.82	-213.8	-256.9	-241.1	-305.3*	-386.6**	-379.9**	-0.0484**
	(163.9)	(161.0)	(168.2)	(169.0)	(166.5)	(179.5)	(180.6)	(0.0224)
Average Stories Per Building						40.10	39.34	0.00458
						(28.86)	(29.52)	(0.00342)
Number of Elevators						42.05	36.46	0.00475
						(61.18)	(61.54)	(0.00742)
Scattered Site						-287.4*	-286.2*	-0.0383*
						(172.6)	(171.2)	(0.0203)
Concrete						67.70	78.98	0.00370
						(226.8)	(228.7)	(0.0274)
Steel						270.7	280.4	0.0323
						(190.6)	(192.1)	(0.0229)
Heat Individually Metered						-556.8**	-539.8**	-0.0693**
						(250.8)	(252.8)	(0.0325)
Electricity Individually Metered						-414.1**	-404.1**	-0.0480**
						(168.0)	(170.0)	(0.0201)
Gas Heat						-187.0	-191.9	-0.0274
						(173.5)	(173.9)	(0.0218)
Electric Heat						-218.5	-309.8	-0.0563
						(297.2)	(302.1)	(0.0390)
Services		208.0	205.9	235.3*	205.9	252.6*	244.4*	0.0272
		(144.3)	(142.0)	(137.6)	(140.6)	(139.5)	(141.3)	(0.0174)
Security		734.6***	754.5***	814.5***	813.5***	820.3***	825.1***	0.107***
		(141.6)	(141.4)	(141.7)	(141.2)	(138.9)	(139.3)	(0.0175)
Constant	5,772***	5,477***	5,414***	6,966***	6,885***	8,230***	8,669***	9.022***
	(345.1)	(345.3)	(564.0)	(737.2)	(742.7)	(987.8)	(1,016)	(0.130)
Observations	439	439	439	439	439	426	426	426
R-squared	0.455	0.487	0.500	0.504	0.502	0.536	0.539	0.534

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1